Improving performance of droop-controlled microgrids through distributed PI-control

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Microgrids facilitate transition to distributed power generation

- Networks with distributed generation (DG) units and loads
- Local, autonomous operation
- Grid-connected or islanded mode

- Synchrony and power balance rely on control of DG units' inverters
- *Droop control:* Decentralized proportional control:

$$\tau_i \dot{\omega}_i = -\omega_i + \omega^{\text{ref}} - k_i (P_i - P_i^{\text{ref}})$$

(ω_i frequency, P_i power injection, τ_i time constant, k_i droop gain)

  - Achieves stability and power sharing
  - Causes stationary frequency error $\omega^{\text{ss}} \neq \omega^{\text{ref}}$

Secondary control eliminates stationary frequency errors

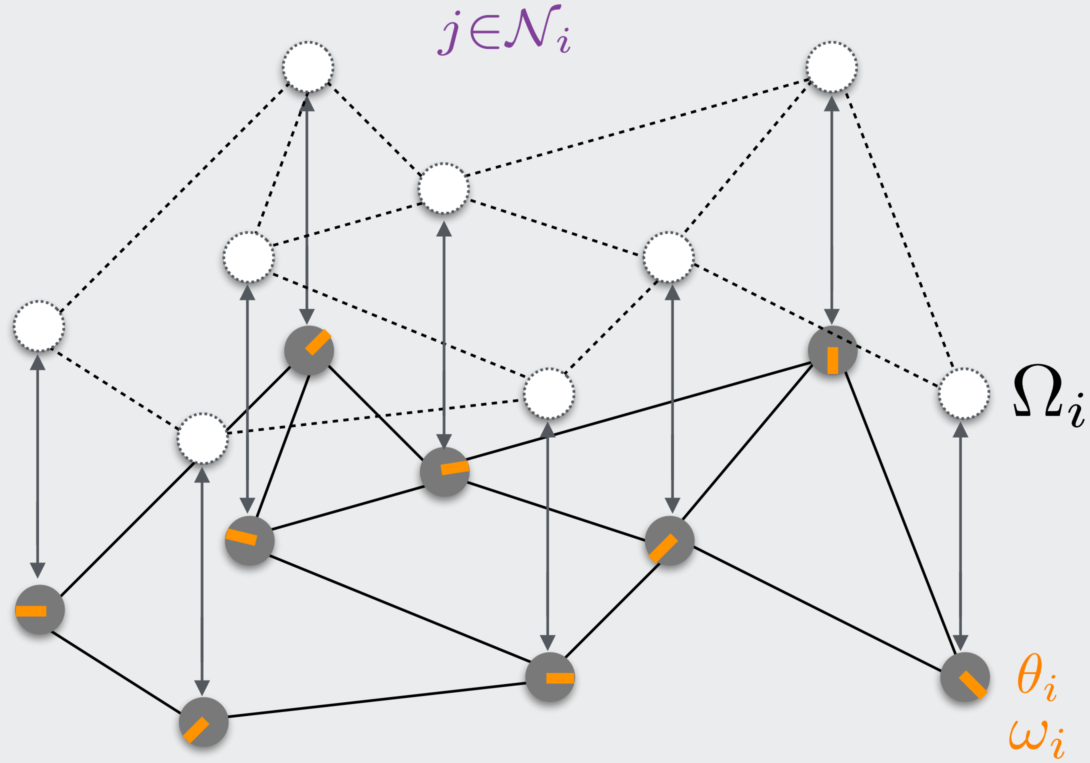
- Naïve approach: Decentralized proportional-integral (PI) control
  Not feasible without PMUs!

$$\dot{\omega}_i = [\text{droop control}] - \frac{1}{q}\int_0^t \omega_i(\tau)d\tau$$

- Distributed averaging PI (DAPI)

$$\dot{\omega}_i = [\text{droop control}] - \Omega_i$$

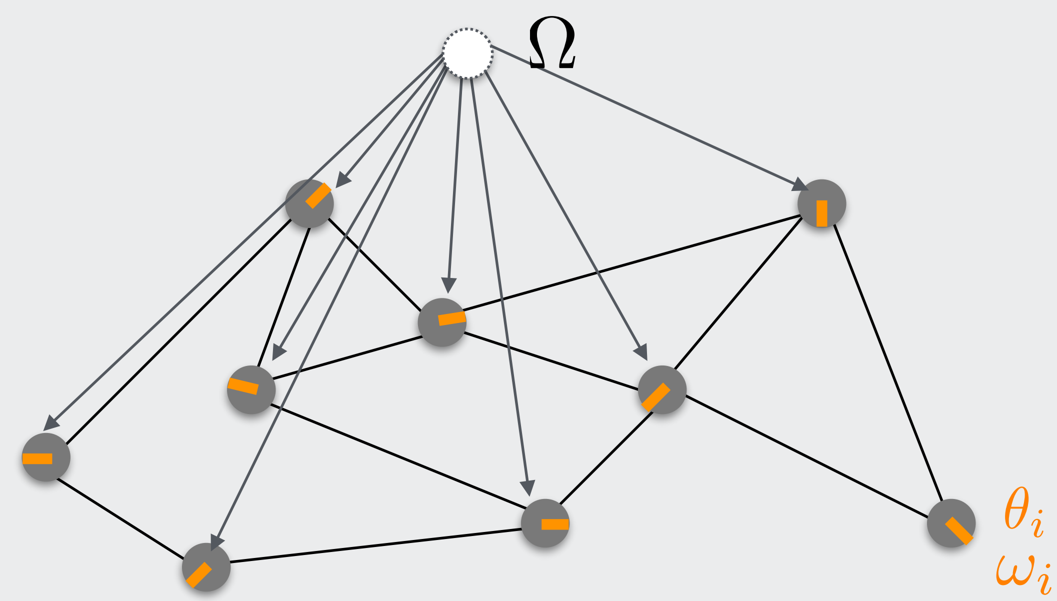
$$q\dot{\Omega}_i = -\omega_i - \sum_{j\in\mathcal{N}_i} c_{ij}(\Omega_i - \Omega_j)$$



- Centralized averaging PI (CAPI)

$$\dot{\omega}_i = [\text{droop control}] - \Omega$$

$$q\dot{\Omega} = -\frac{1}{N}\sum_{i\in\mathcal{V}} \omega_i$$

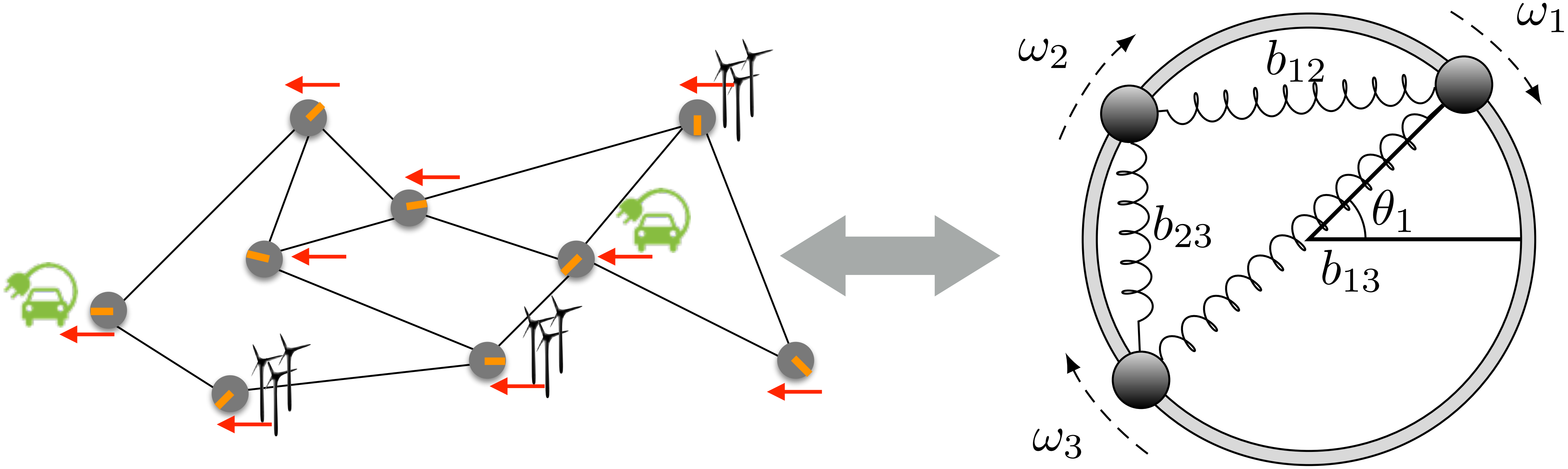


(q, c_{ij} constant gains, $\mathcal{N}_i$ neighbor set of node i, $\mathcal{V}$ set of all nodes)

Related work has focused on stability and power sharing with DAPI and CAPI

- See, for example:
  - J. W. Simpson Porco, F. Dörfler and F. Bullo: “Synchronization and power sharing for droop-controlled inverters in islanded microgrids,” *Automatica* 2013.
  - M. Andreasson, D. Dimarogonas, H. Sandberg, and K. H. Johansson, “Distributed control of networked dynamical systems: Static feedback, integral action and consensus,” *IEEE TAC* 2014
  - C. Zhao, E. Mallada, and F. Dörfler, “Distributed frequency control for stability and economic dispatch in power networks,” ACC 2015
  - F. Dörfler, J. W. Simpson-Porco, and F. Bullo, “Breaking the Hierarchy: Distributed Control & Economic Optimality in Microgrids,” *IEEE TCNS, 2015*
- This work: focus on *performance*

Performance is measured through transient power losses



Analogy due to F. Dörfler et al.

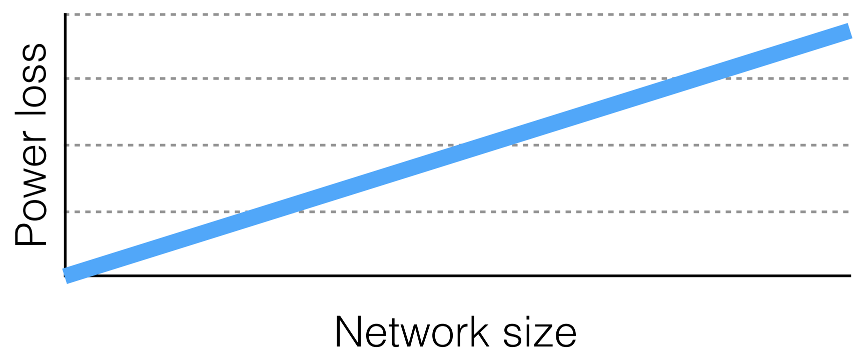
Performance measure: *cost* of maintaining synchrony

- Assume stable operating conditions
- Assume distributed stochastic disturbances
- Quantify power losses during re-synchronization
- Related to *coherence measures* (c.f. Bamieh *et al.*, *IEEE TAC* 2012)

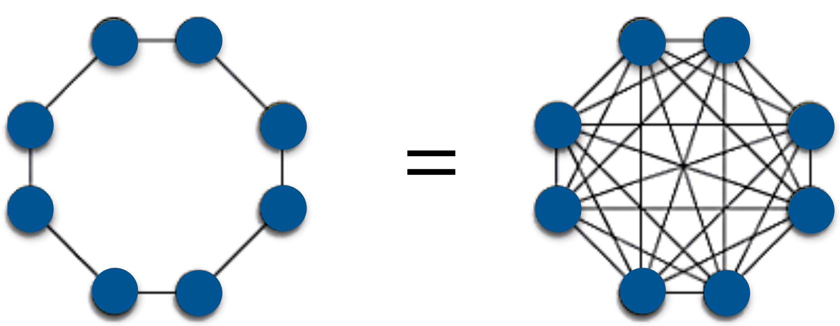
Distributed PI control improves performance compared to droop control

Previous results: Performance with primary (droop) control

- Linear growth with network size



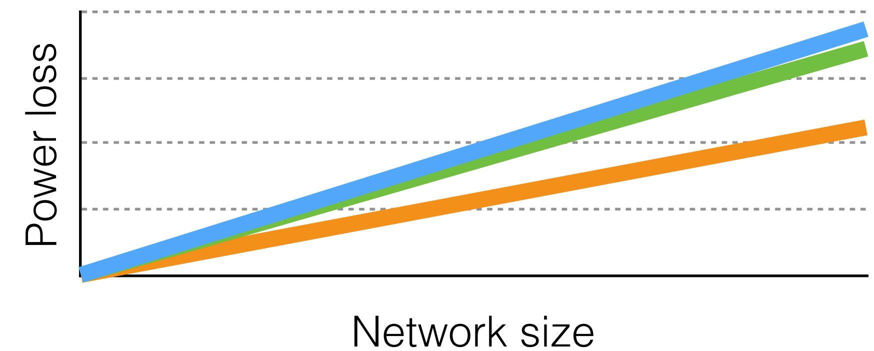
- Independent of connectivity



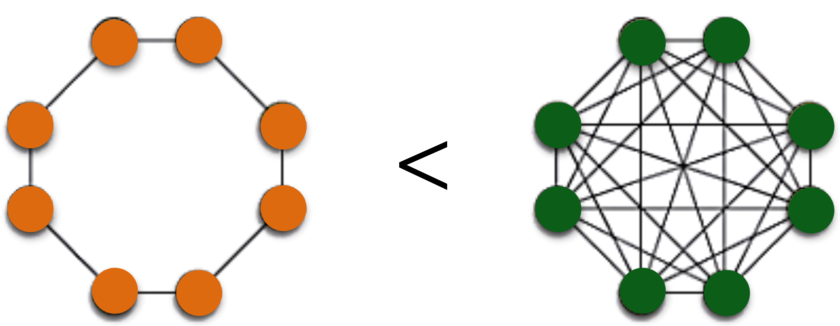
Bamieh & Gayme, ACC 2013
Tegling et al. IEEE TCNS 2015

Today: Performance with secondary control (DAPI)

- DAPI gives strictly smaller losses than droop control



- Smaller losses in sparse networks

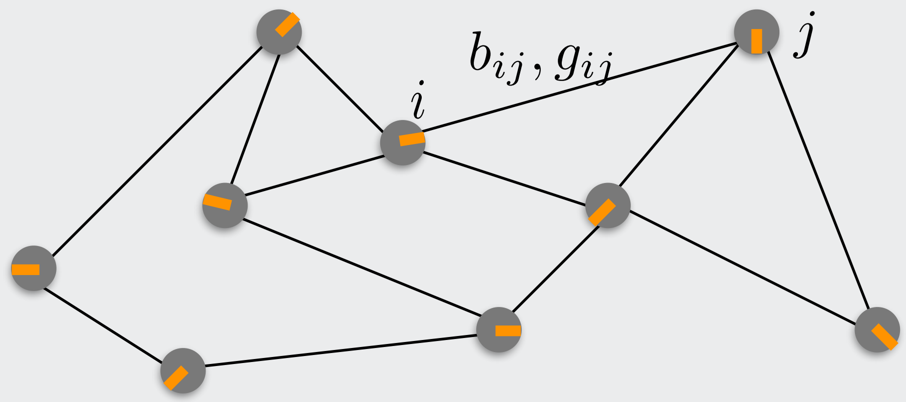


OUTLINE

- Introduction and problem formulation
- Model of droop-controlled microgrid
- Performance evaluation
- Control design for loss reduction
- Conclusions and directions for future work

Microgrid modeled as LTI system subject to disturbances 1(2)

- Network: N-node graph representing AC power lines between inverters.
- Weighted graph Laplacians L_B, L_G
  - Susceptance matrix L_B, weights b_{ij}
  - Conductance matrix L_G weights g_{ij}



- DAPI control law:

$$\dot{\theta}_i = \omega_i$$

$$\tau_i \dot{\omega}_i = -\omega_i + \omega^{\text{ref}} - k_i(P_i - P_i^{\text{ref}}) + \Omega_i$$

$$q\dot{\Omega}_i = -\omega_i + \omega^{\text{ref}} - \sum_{j \in \mathcal{N}_i} c_{ij}(\Omega_i - \Omega_j)$$

  - Standard droop control: $q \to \infty$
  - CAPI control: $c_{ij} \to \infty$

($\mathcal{N}_i$ neighbor set of node i, c_{ij} constant gains, here $c_{ij} = \gamma b_{ij}$, q integral gain)

- Linear power flow: Power injection at node i

$$P_i = \sum_{j \in \mathcal{N}_i} b_{ij} \left(\theta_i - \theta_j\right)$$

(b_{ij} line susceptance)

Microgrid modeled as LTI system subject to disturbances 2(2)

- Closed loop system:
  - Represents deviations from operating point
  - Distributed disturbances due to e.g. generation/load fluctuations

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -\frac{k}{\tau} L_B & -\frac{1}{\tau} I & \frac{1}{\tau} I \\ 0 & -\frac{1}{q} I & -\frac{1}{q}\gamma L_B \end{bmatrix} \begin{bmatrix} \theta \\ \omega \\ \Omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\tau} I \\ 0 \end{bmatrix} w$$

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Power losses measured through appropriate performance output

Recall: H_2 norm for general system under white noise input:

$$||H||_2^2 = \lim_{t\to\infty} \mathbb{E}\{y^*(t)y(t)\}$$

Idea: define system output so that $\mathbf{P}^{\text{loss}}(t) = y^*(t)y(t)$

- Power loss over line $i{,}j$ (Ohm's law, quadratic approximation):

$$P_{ij}^{\text{loss}}(t) \approx g_{ij}(\theta_i - \theta_j)^2$$

(g_{ij} line conductance)

- Total losses over network:

$$\mathbf{P}^{\text{loss}} = \sum_{e_{ij}\in\mathcal{E}} g_{ij}(\theta_i - \theta_j)^2 = \theta^* L_G \theta$$

- Set performance output to:

$$y(t) = L_G^{1/2}\theta(t)$$

Expected losses now given by the system's H_2 norm

CAPI leaves performance unchanged, while DAPI reduces losses

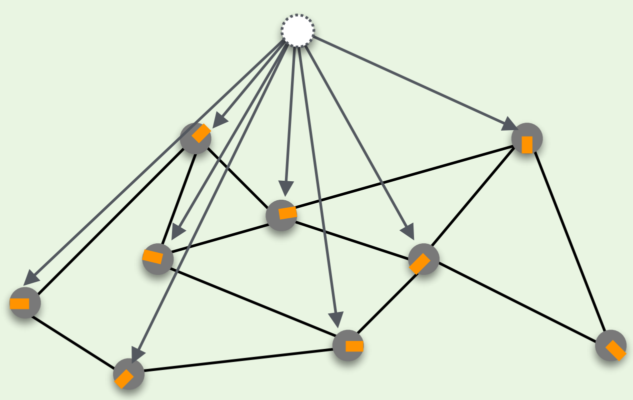
- Assume uniform conductance-to-susceptance ratios: $\frac{g_{ij}}{b_{ij}} = \alpha$
- Assume uniform droop gains $k_i = k$

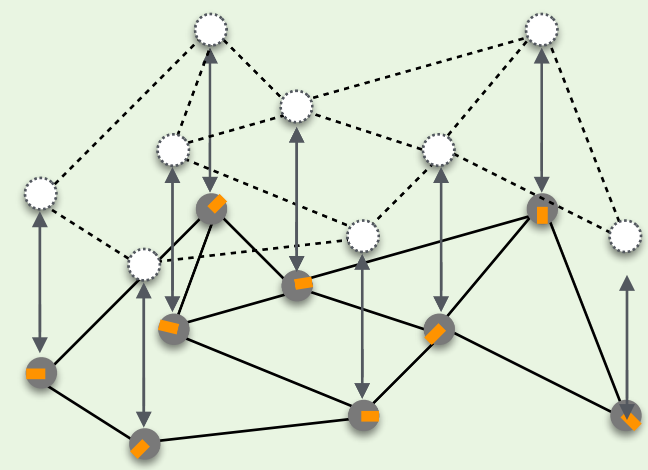
Theorem

$$||H_{\text{CAPI}}||_2^2 = \frac{\alpha}{2k}(N-1)$$
$$= ||H_{\text{droop}}||_2^2$$

$$||H_{\text{DAPI}}||_2^2 = \frac{\alpha}{2k}\sum_{n=2}^{N} \frac{1}{1 + \frac{\gamma\tau\lambda_n + q}{\gamma\lambda_n(\gamma\tau\lambda_n + q) + q^2 m\lambda_n}}$$

N-1 terms, each<1





Corollary

$$||H_{\text{DAPI}}||_2^2 < ||H_{\text{CAPI}}||_2^2 = ||H_{\text{droop}}||_2^2$$

(k droop gain, N network size, $\gamma = c_{ij}/b_{ij}$, where c_{ij} is distr. averaging gain, q integral gain, λ_n eigenvalues of susceptance matrix L_B)

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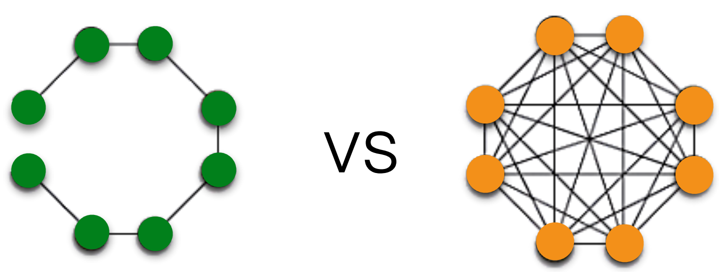
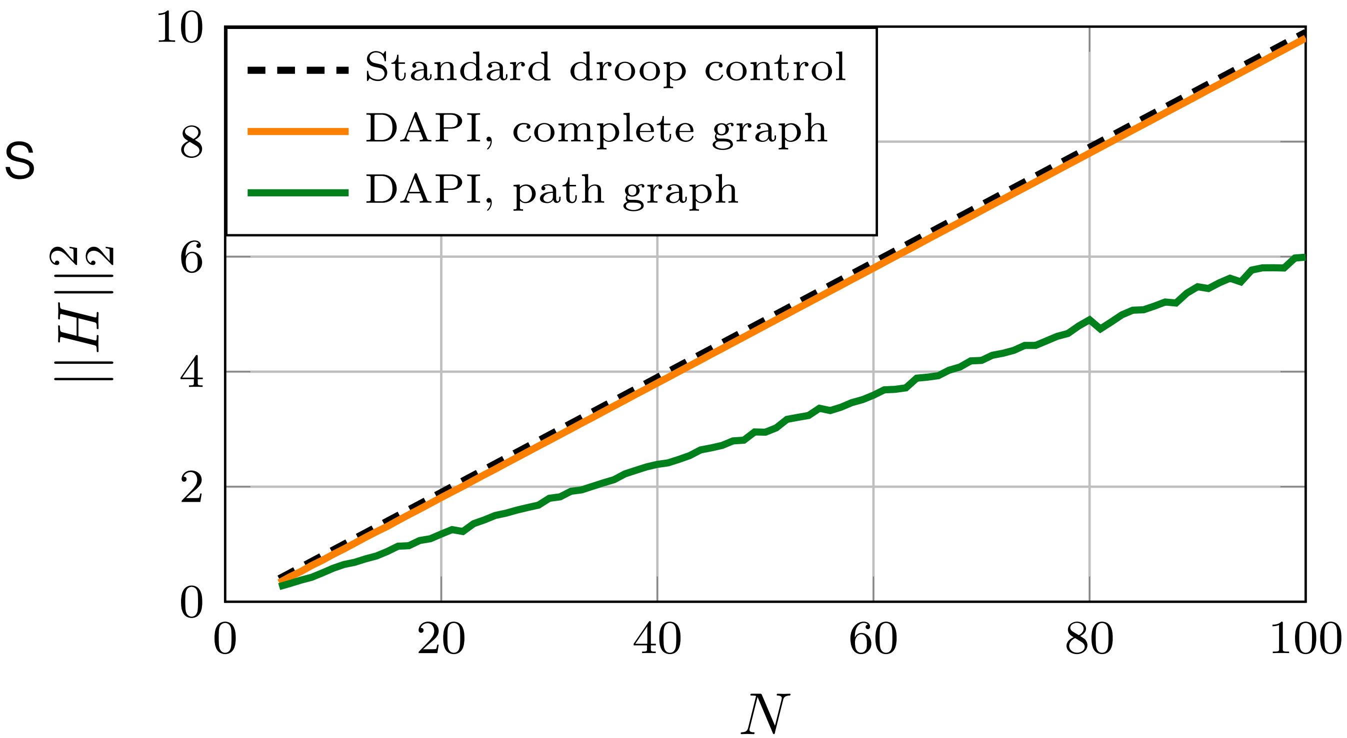
Better performance with sparse topology and little alignment

Network topology dependence

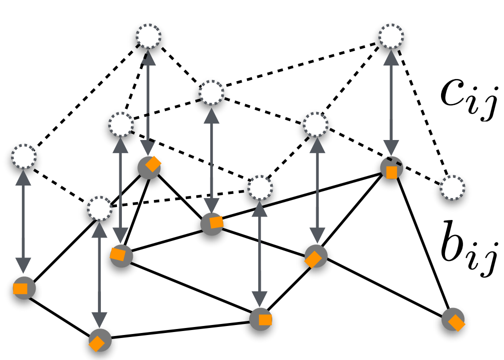
- Smaller losses for sparse topologies
- Losses scale with network size N



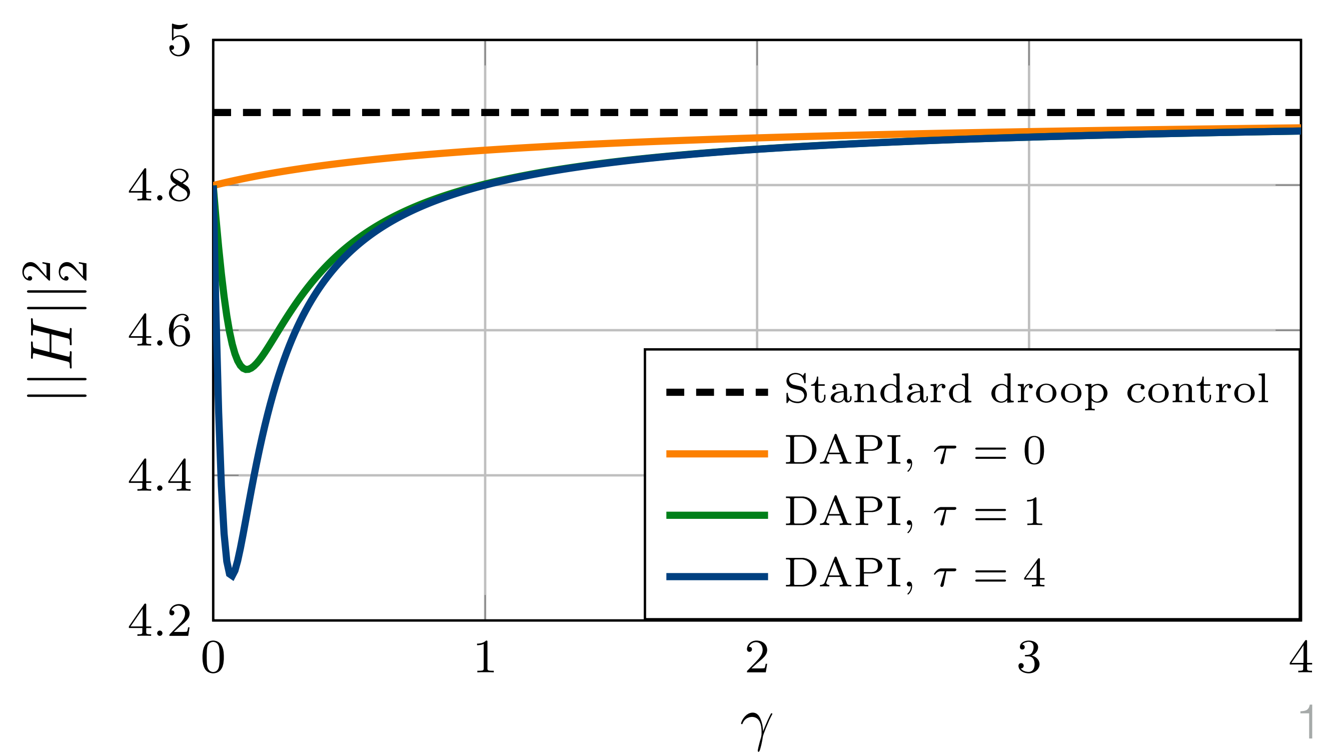


Optimal gain for distributed averaging

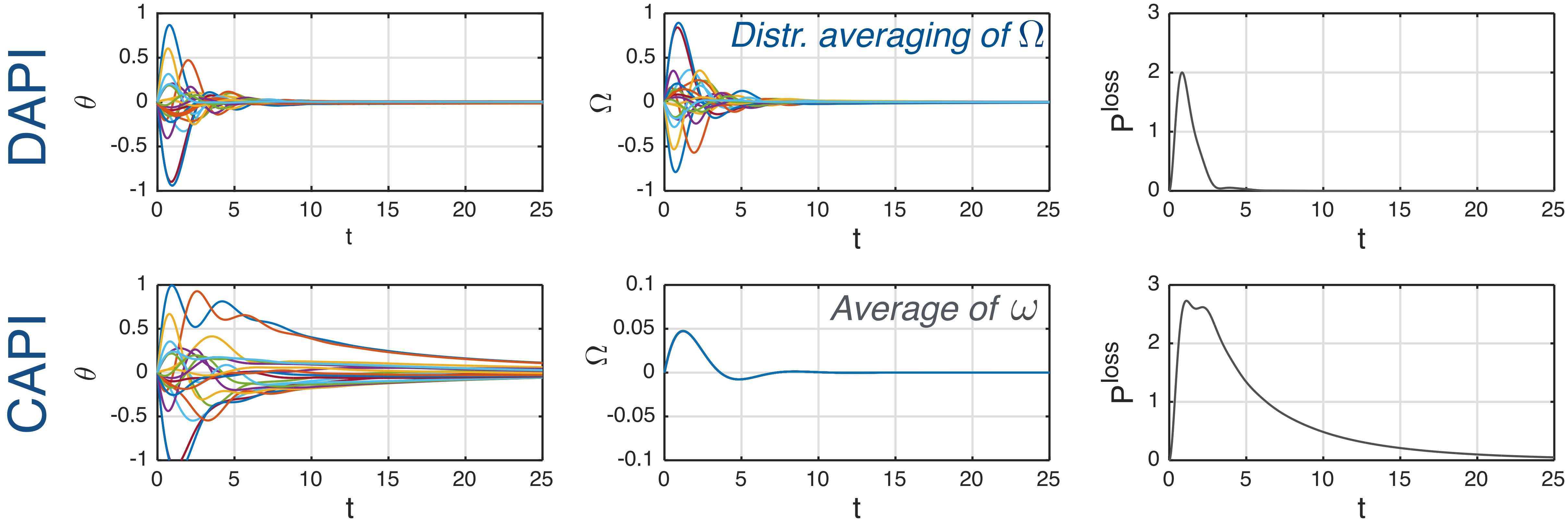
- Distinct optimal parameter $\gamma^\star$
- $\gamma^\star$ often very small
- Recall, $\gamma = 0$ not feasible!



$\gamma = c_{ij}/b_{ij}$



Self-damping key to improved performance



Synchronization transients in 20 node network

- DAPI provides substitute for self-damping on phase angles

$$\dot{\omega}_i \approx [\text{droop control}] - \frac{1}{q}\underbrace{\int_0^t \omega_i(\tau)d\tau}_{\to \theta_i}$$

- "Cheaper" to rely on self-damping, as power flows associated with costs
- Need to align with neighbors — but too strong alignment reduces self-damping effect

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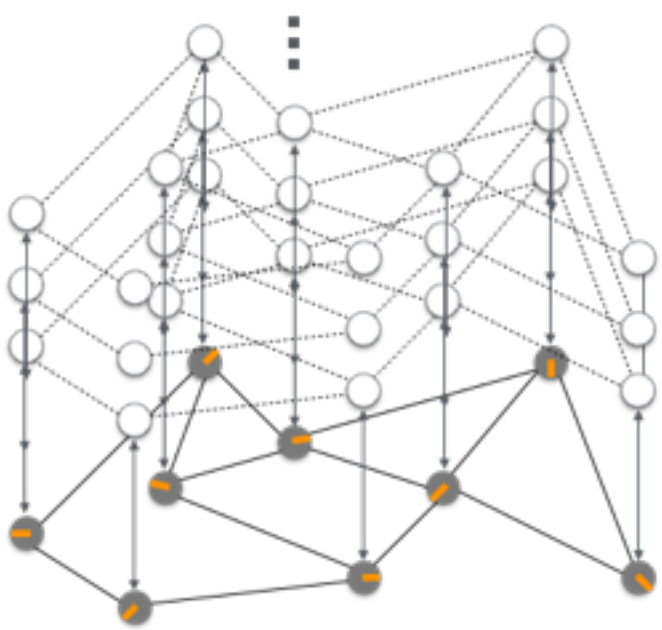
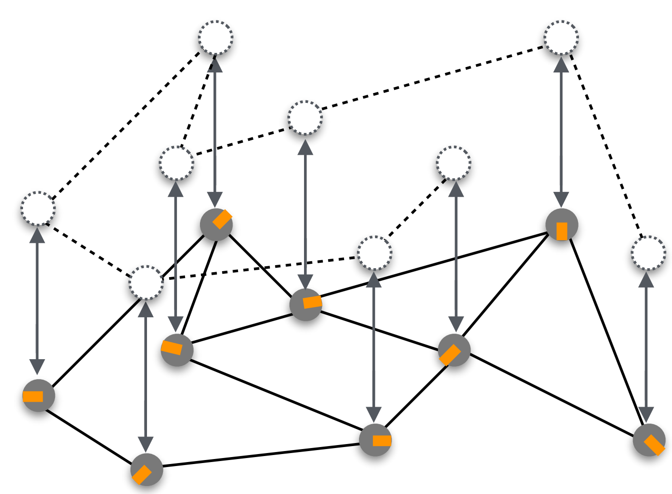
Concluding summary: Results show benefits of distributed algorithms in microgrids

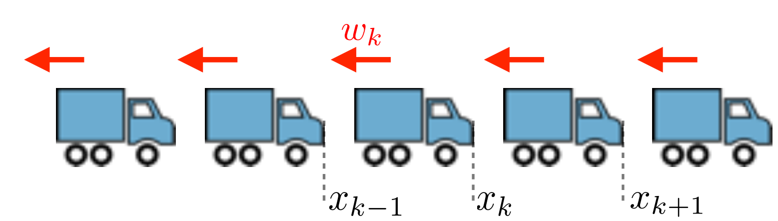
- Evaluated performance of inverter-based microgrids through H_2 norm
- Compared standard droop control to PI control with distributed and centralized averaging (DAPI and CAPI)
- DAPI control improves performance by emulating self-damping
- Better performance in sparse topologies, with small-gain averaging between nodes

Ongoing and future work: further analysis of distributed dynamic feedback

- Higher-order controllers
- Topology of secondary controller layer
- Other applications and performance metrics (e.g. coherence in large-scale vehicular formations)







Thank you!

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