On performance limitations of large-scale networks with distributed feedback control

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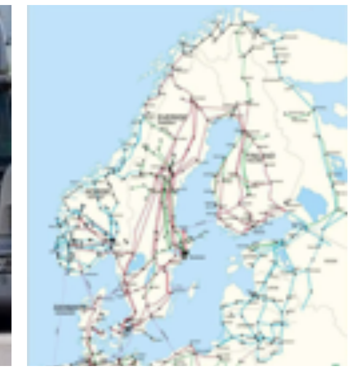
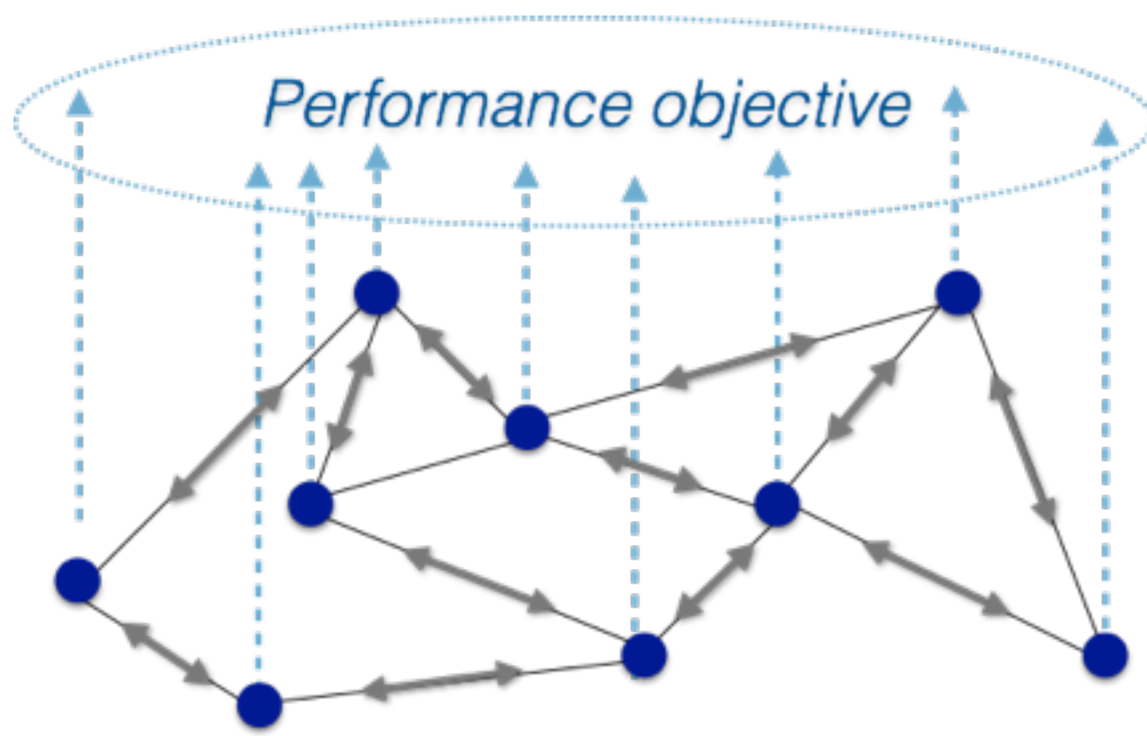
Networked systems: *global* objectives, but *local* feedback

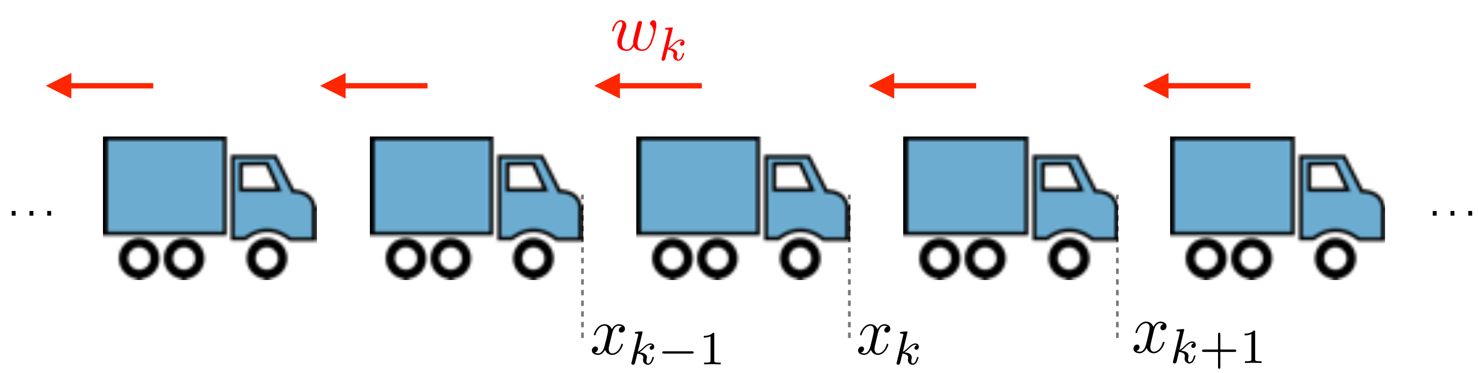






Are there limitations to network *performance*?

Example 1: Vehicle platoons can reduce emissions and increase road throughput



- Objectives:
  - common cruising speed
  - tight constant spacing Δ

- Dynamics (example): look-ahead, look-behind control

$$\ddot{x}_k = \dot{v}_k = f_+(x_{k+1} - x_k - \Delta) + f_-(x_{k-1} - x_k - \Delta) + g_+(v_{k+1} - v_k) + g_-(v_{k-1} - v_k) + w_k$$

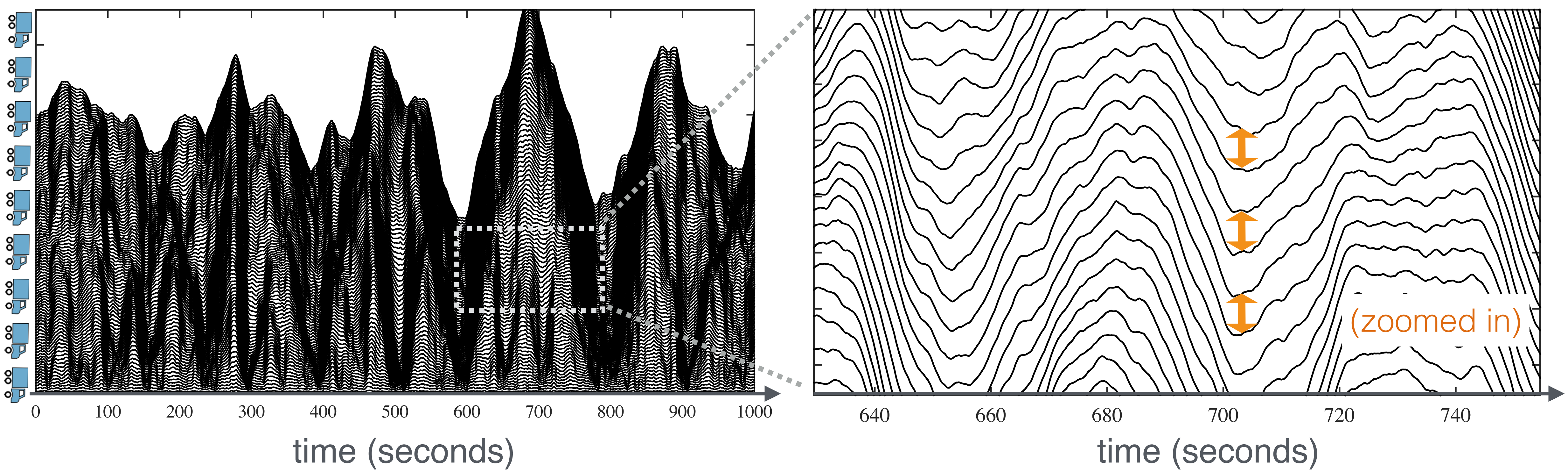
(f_+, f_-, g_+, g_- constant gains)

- With disturbances: objectives only achieved approximately

- What happens if the platoon grows?

Example 1: Performance issues if control is based on *relative* measurements

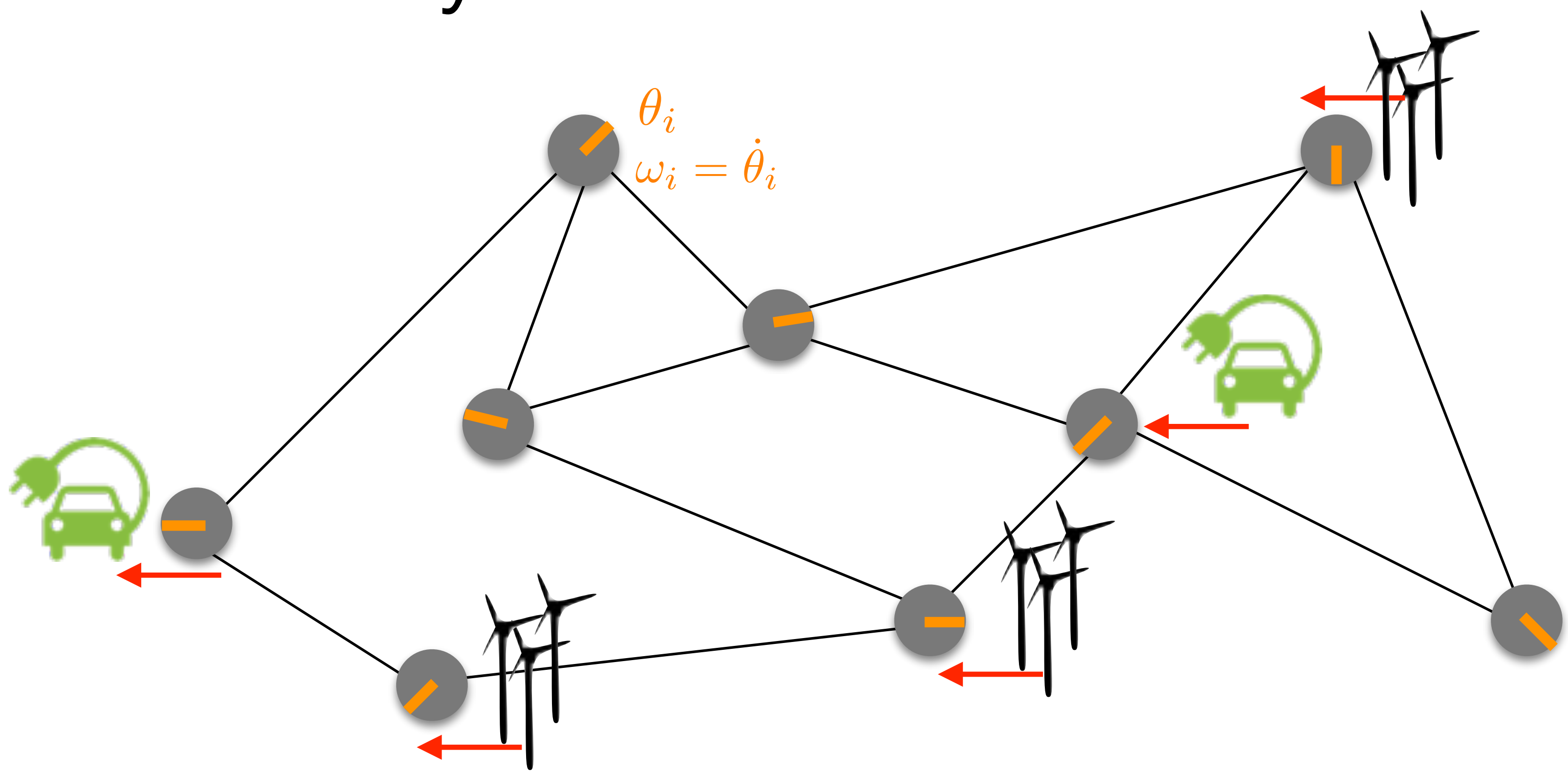
Time trajectories of 100 vehicles, relative to leader, seen from above



- Formation is stable
- Spacings ⟷ are well-regulated (no collisions!)
- However - not a *rigid* formation, not *coherent!*
- Fundamental limitation to local, static feedback (Bamieh *et al.*, 2012)

Can *dynamic* control laws help?

Example 2: Transition to a greener power system affects network synchronization



- Objectives:
  - common, steady frequency (50 Hz)
  - phase angles at equilibrium
- More disturbances due to
  - renewable, intermittent generation
  - changing load patterns
- Networks grow as generation becomes distributed

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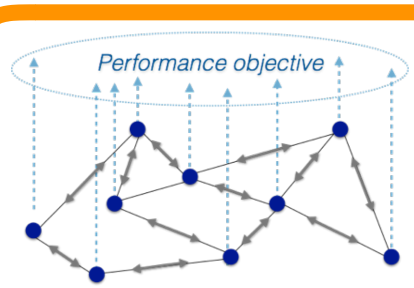
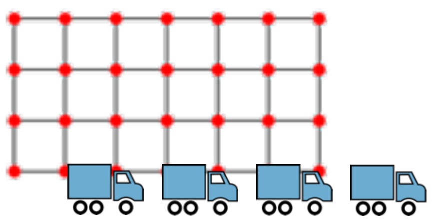


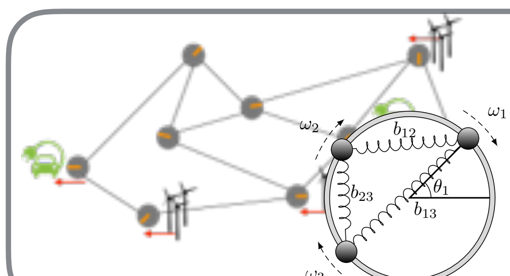
Introduction and problem formulation

$\mathcal{H}_2$

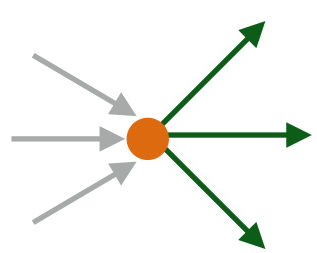
Evaluating input-output performance



Case 1: Regular lattice networks, coherence

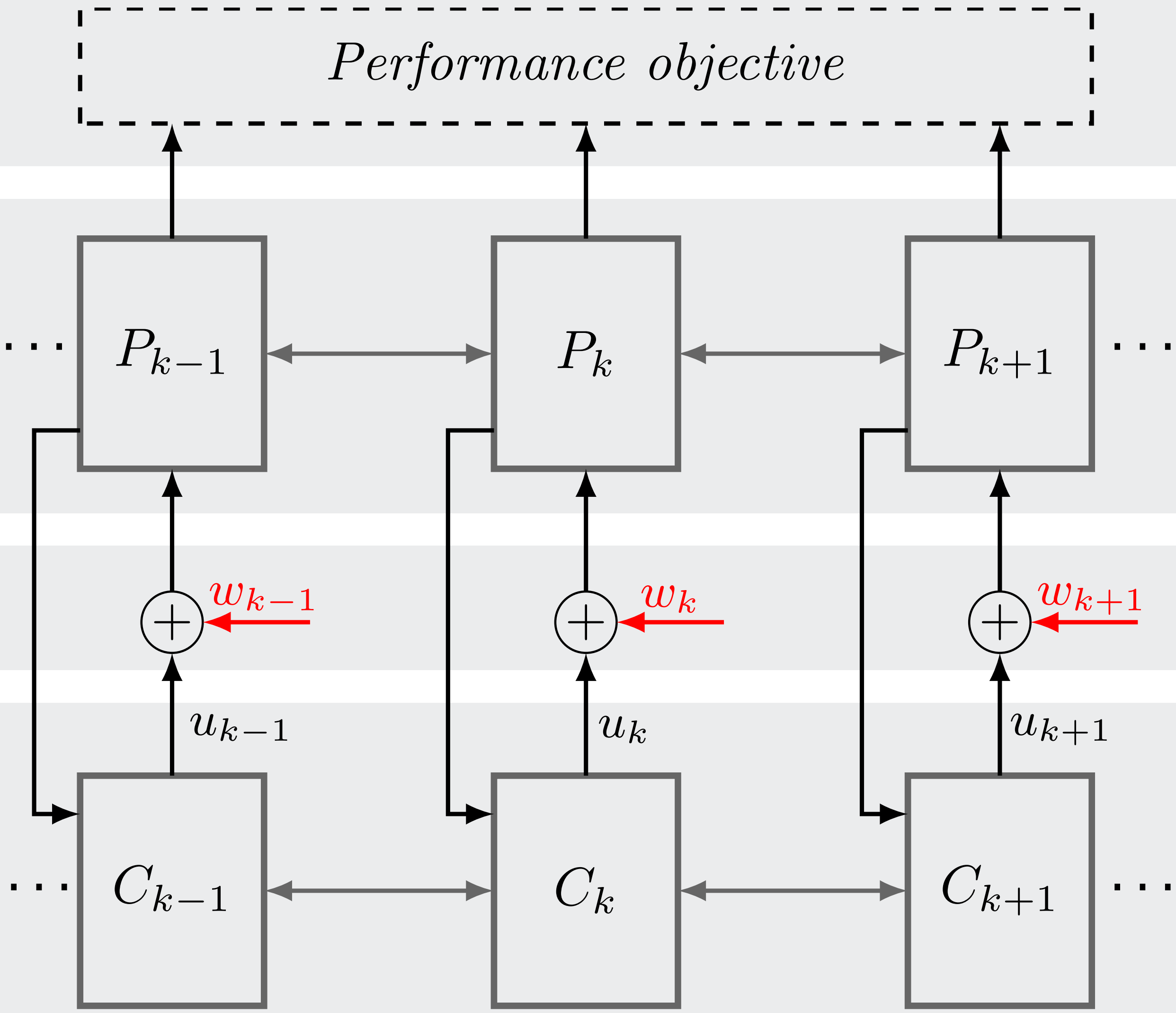


Case 2: Power networks, price of synchrony



Conclusions and future work

Setup: The distributed control problem



- Plant: Single / double integrator

  $\dot{x}_k = u_k + w_k \quad / \quad \ddot{x}_k = u_k + w_k$

- Additive white noise

- Control law: consensus dynamics

$$u_k = \sum_{j \in \mathcal{N}_k} f_{jk}(x_j - x_k) + \sum_{j \in \mathcal{N}_k} g_{jk}(\dot{x}_j - \dot{x}_k)$$

Relative feedback from x, $\dot{x}$

$$-f_o x_k - g_o \dot{x}_k$$

Absolute feedback (/self-damping)

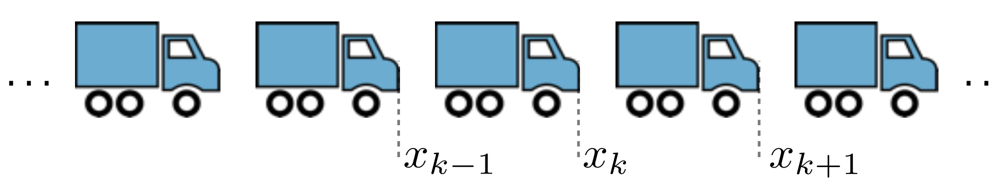
Setup: Performance is evaluated through global and local measures of "disorder"

Global error
- *coherence*

- Deviation from network average

$$y_k^{dav} = x_k - \frac{1}{N}\sum_{j=1}^{N} x_j$$

- Characterizes rigidity, coherence

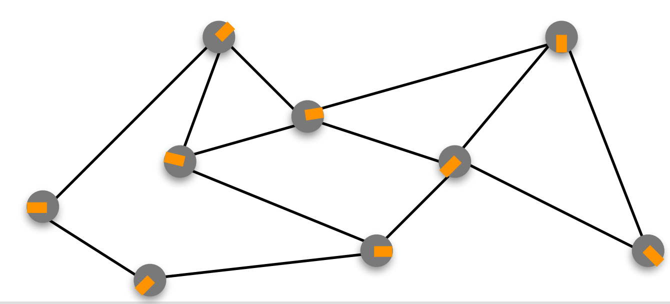


Local error
- *lack of synchrony*

- Deviation from neighbor average

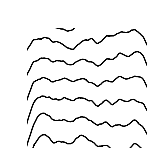
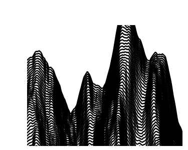
$$y_k^{loc} = \sum_{j \in \mathcal{N}_k} a_{j,k}(x_k - x_j)$$

- Characterizes lack of local order, or synchrony

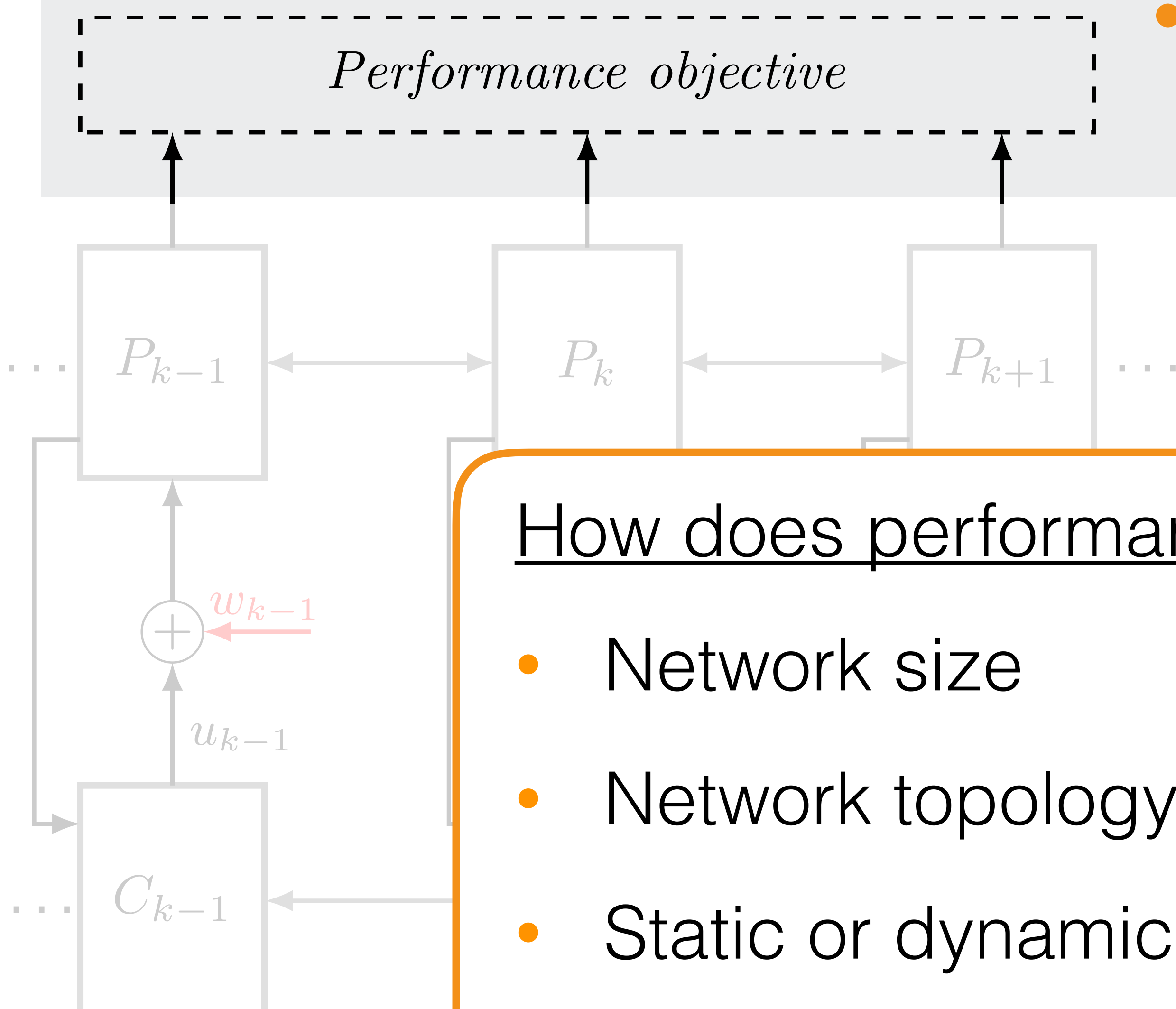


- Performance is measured through *variance*:

$$V_k = \mathbb{E}\left\{ (y_k)^2 \right\}$$

- Note! Two distinct performance measures (c.f.  vs )

Objective: characterize performance in large-scale networks



- Performance: $V_k = \mathbb{E}\left\{(y_k)^2\right\}$, where $y_k = \text{global error}$, or $y_k = \text{local error}$

<u>How does performance depend on:</u>

- Network size
- Network topology / controller structure
- Static or dynamic feedback

What fundamental limitations are there?

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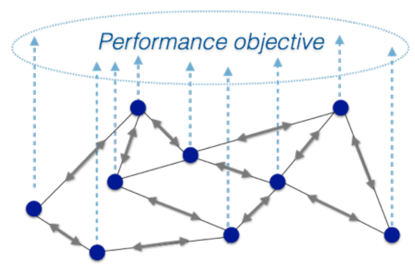

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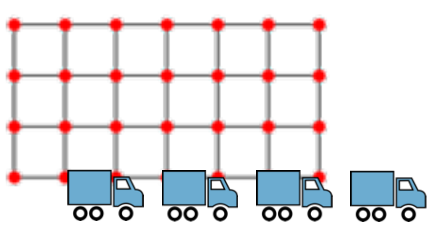

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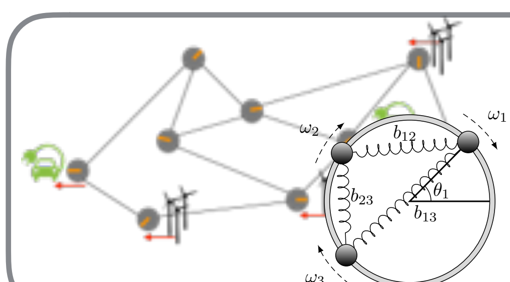

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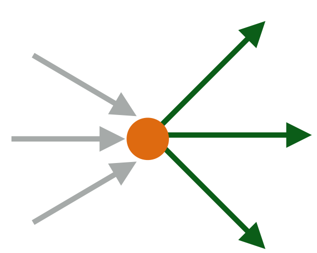

Conclusions and future work

Performance is evaluated through input-output H_2 norms

Consider general linear system under white noise input

$$\dot{x} = \mathcal{A}x + \mathcal{B}w$$
$$y = \mathcal{C}x \qquad (1)$$

Recall:

Need to evaluate $V_k = \mathbb{E}\{(y_k)^2\}$, with e.g. $y_k = x_k - \frac{1}{N}\sum_{j=1}^{N} x_j$.

Lemma:

The squared H_2 norm of (1) from input w to output y gives

$$||H||_2^2 = \lim_{t\to\infty} \mathbb{E}\{y^*(t)y(t)\},$$

That is, the steady state output variance.

➡ With the appropriate output, performance is given by H_2 norm!

Evaluating system performance amounts to evaluating H_2 norms!

Unitary transformation simplifies H$_2$ norm evaluation 1(2)

$$\hat{H}: \quad \dot{\hat{x}} = \underbrace{\begin{bmatrix} \ddots & \ddots \\ \ddots & \ddots \end{bmatrix}}_{\hat{\mathcal{A}}_{N\times N}} \hat{x} + \underbrace{\begin{bmatrix} \ddots \\ \ddots \end{bmatrix}}_{\hat{\mathcal{B}}} \hat{w}$$

$$\hat{y} = \underbrace{\begin{bmatrix} \ddots & \ddots \end{bmatrix}}_{\hat{\mathcal{C}}} \hat{x}$$

$$\hat{H}_n: \quad \dot{\hat{x}}_n = \hat{\mathcal{A}}_n \hat{x}_n + \hat{\mathcal{B}}_n \hat{w}_n$$

$$\hat{y}_n = \hat{\mathcal{C}}_n \hat{x}_n$$

- Unitary transformation does not change H$_2$ norm
- (Block-) diagonalize to obtain N decoupled subsystems $\hat{H}_n$
- H2 norm is sum of subsystem norms:

$$||H||_2^2 = ||\hat{H}||_2^2 = \sum_{n=1}^{N} ||\hat{H}_n||_2^2$$

- *Zero mode* associated with *drift of average* makes $\hat{\mathcal{A}}_1$ non-Hurwitz.
- Mode *unobservable*, so $||\hat{H}_1||_2^2 = 0$.
- Only sum over remaining, stable, subsystems: $||H||_2^2 = \sum_{n=2}^{N} ||\hat{H}_n||_2^2$

Unitary transformation simplifies H_2 norm evaluation 2(2)

H_2 norm evaluated as sum of subsystem norms: $||H||_2^2 = \sum_{n=2}^{N} ||\hat{H}_n||_2^2$

- For n such that $\hat{\mathcal{A}}_n$ Hurwitz, $||\hat{H}_n||_2^2 = \mathrm{tr}\left(\hat{\mathcal{B}}_n^* X_n \hat{\mathcal{B}}_n\right)$, where

$$\hat{\mathcal{A}}_n^* X_n + X_n \hat{\mathcal{A}}_n = -\hat{\mathcal{C}}_n^* \hat{\mathcal{C}}_n. \quad (1)$$

Example:

Assume $\hat{\mathcal{A}} = -\mathrm{diag}\{\lambda_n^{\mathcal{A}}\}$, then $\hat{\mathcal{A}}_n = -\lambda_n^{\mathcal{A}}$ is scalar. Here, $\hat{\mathcal{B}}_n = 1$.

Then (1) gives $$||\hat{H}_n||_2^2 = X_n = \frac{\hat{c}_n^2}{2\lambda_n^{\mathcal{A}}}$$

=1 in Part 1

Eigenvalue of $\mathcal{A}$!

so $||H||_2^2 = \frac{1}{2}\sum_{n=2}^{N} \frac{\hat{c}_n^2}{\lambda_n^{\mathcal{A}}}$.

H_2 norms involve sums over inverted eigenvalues, $\sum_{n=2}^{N} \frac{1}{\lambda_n^{\mathcal{A}}}$!

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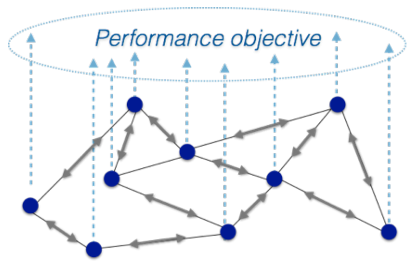

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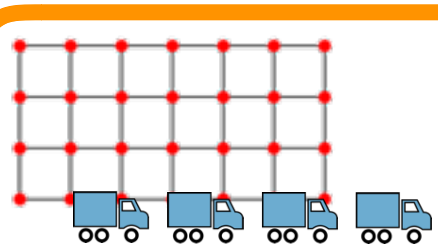

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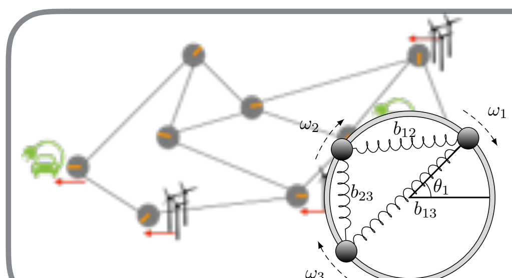

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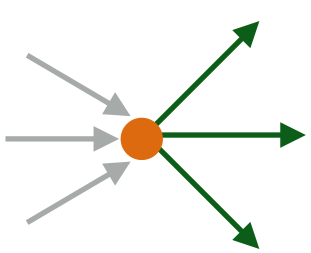
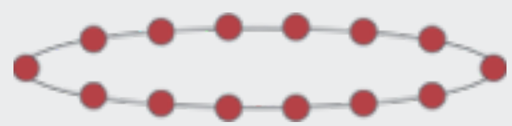


Conclusions and future work

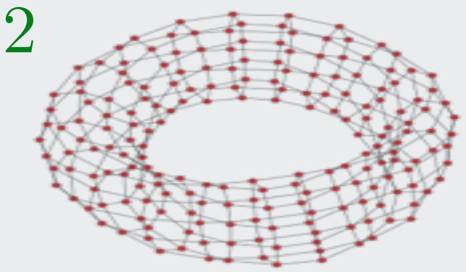
Consensus and vehicular formation problems modeled over toric lattices

- Network: d-dimensional discrete torus $\mathbb{Z}_N^d$. Network size: $M = N^d$.

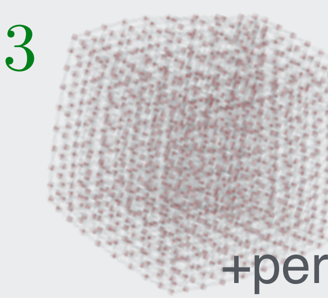
$d = 1$



$d = 2$



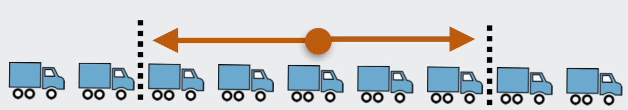
$d = 3$



+periodicity

- Dynamics:
  - Consensus (1st order) $\dot{x}_k = u_k + w_k$
  - Vehicular formations (2nd order) $\ddot{x}_k = \dot{v}_k = u_k + w_k$

- Control: Standard, static feedback
  - Consensus: $u_k = (Fx)_k$ → $\dot{x} = Fx + w$
  - Vehicular formations: $u_k = (Fx)_k + (Gv)_k$



Only feedback from *local* neighborhood!

→ $$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & I \\ F & G \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w$$

(F, G look like graph Laplacians!)

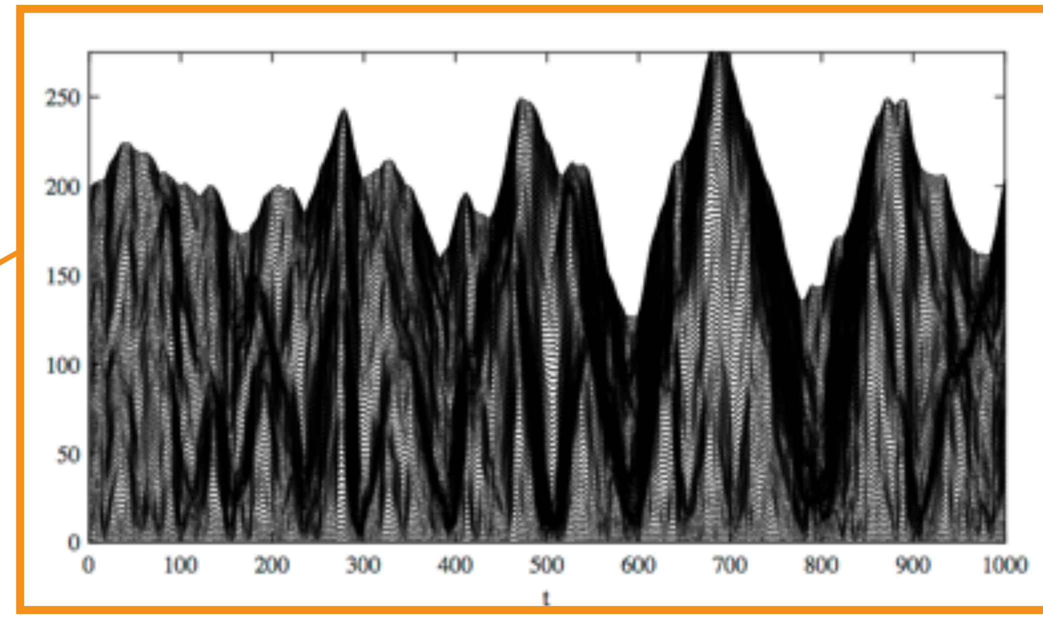
With static feedback: performance scales badly in low lattice dimensions

- Study *asymptotic* scaling of the variance $V_k = \mathrm{var}(x_k - \frac{1}{M}\sum_{j=1}^{M} x_j)$
- The better V_k scales, the more *coherent* the system
- *Fully* coherent if V_k does not grow as network size $M \to \infty$

Asymptotic scalings with *static* feedback (Bamieh *et al.*, 2012)

Up to a constant independent of gain parameter β and network size M

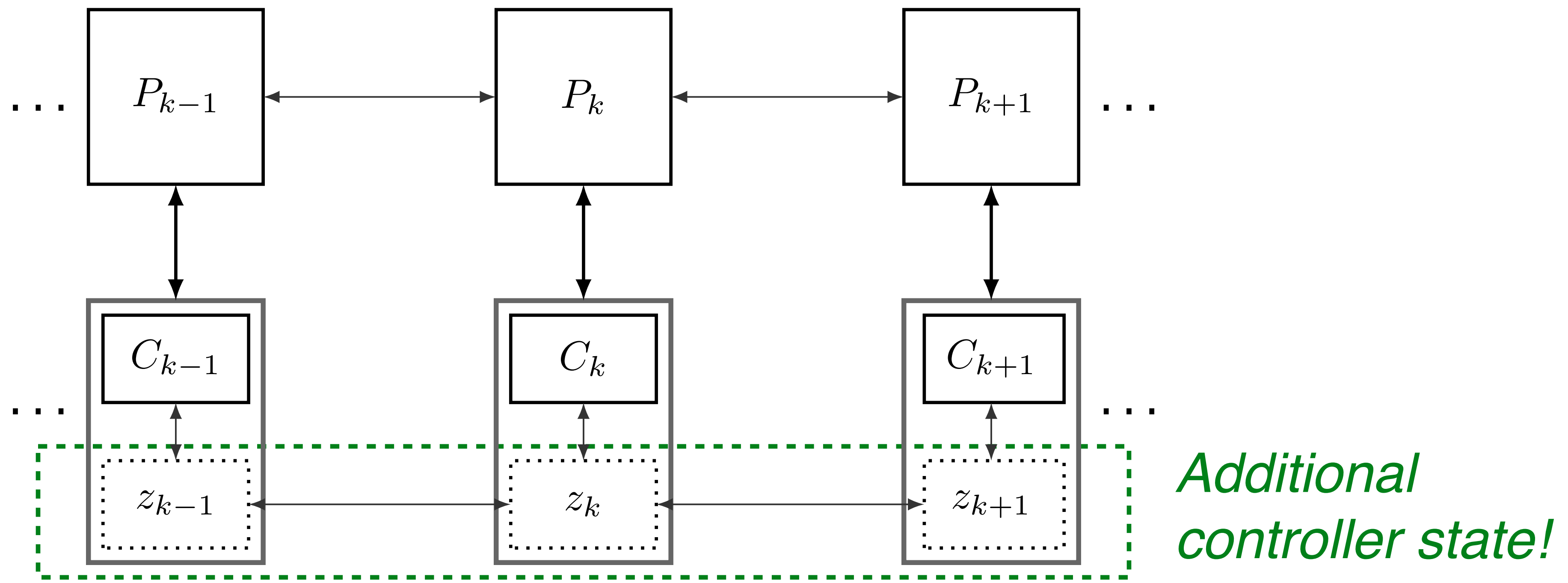
Absolute x, v	$V_k \sim \frac{1}{\beta}$
Relative x, absolute v	$V_k \sim \frac{1}{\beta}\begin{cases} M & d=1 \\ \log M & d=2 \\ 1 & d \geq 3, \end{cases}$
Relative x, relative v	$V_k \sim \frac{1}{\beta^2}\begin{cases} M^3 & d=1 \\ M & d=2 \\ M^{1/3} & d=3 \\ \log M & d=4 \\ 1 & d \geq 5, \end{cases}$



$$u_k = f_+(x_{k+1} - x_k) + f_-(x_{k-1} - x_k) + g_+(v_{k+1} - v_k) + g_-(v_{k-1} - v_k) + f_o x_k + g_o v_k$$

Relative feedback from x — Relative feedback from v — Absolute feedback

Introducing distributed *dynamic* feedback: control with memory



- <u>Proposed control</u>: General dynamic feedback
  - Consensus:

$$u_k = (Fx)_k + z_k$$
$$\dot{z}_k = (Az)_k + (Bx)_k$$

$$\begin{bmatrix} \dot{z} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} A & B \\ I & F \end{bmatrix} \begin{bmatrix} z \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w$$

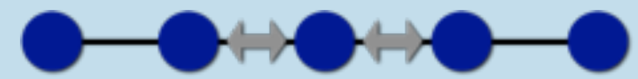
  - Vehicular formations:

$$u_k = (Fx)_k + (Gv)_k + z_k$$
$$\dot{z}_k = (Az)_k + (Bx)_k + (Cv)_k$$

$$\begin{bmatrix} \dot{z} \\ \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} A & B & C \\ 0 & 0 & I \\ I & F & G \end{bmatrix} \begin{bmatrix} z \\ x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} w$$

Bad performance scaling is caused by eigenvalues near zero

Example (Standard consensus, 1st order):



$$\dot{x} = Fx + w$$

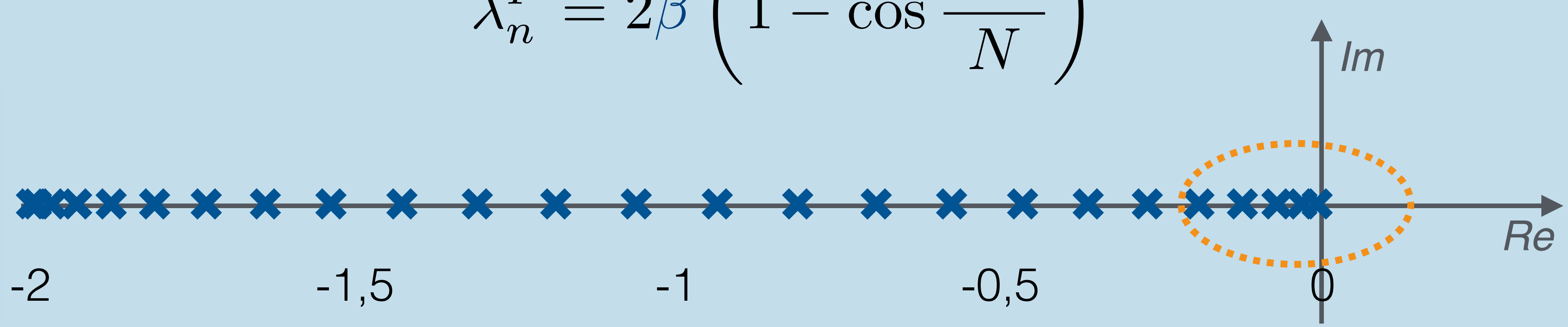
$$\dot{x}_k = \beta(x_{k+1} - x_k) + \beta(x_{k-1} - x_k) + w_k$$

Recall:

$$||H||_2^2 = \frac{1}{2}\sum_{n=2}^{N}\frac{1}{\lambda_n^F}$$

- Eigenvalues

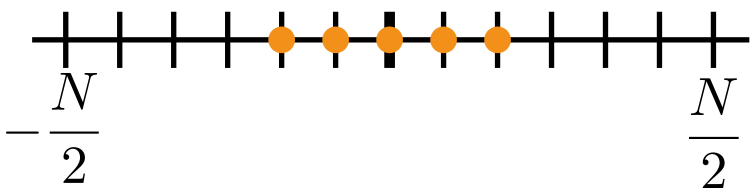
$$\lambda_n^F = 2\beta\left(1 - \cos\frac{2\pi n}{N}\right)$$



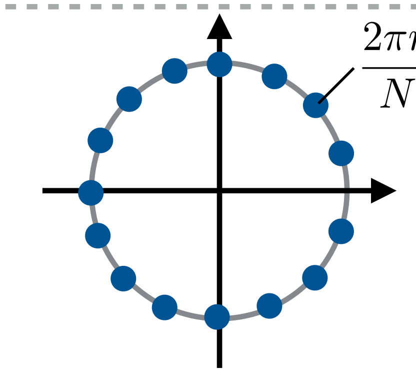
- As N grows: Arbitrarily many λ_n^F increasingly close to zero - sum blows up!

Evaluating performance in the limit from finite to infinite lattices

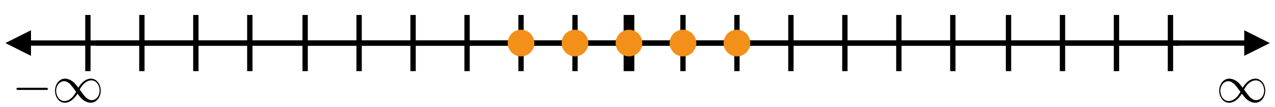
- Eigenvalues λ_n^F ⟷ Spatial discrete Fourier transforms



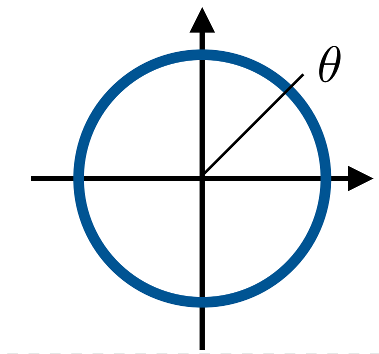
$$\hat{f}_n := \sum_{k \in \mathbb{Z}_N^d} f_k e^{-j \frac{2\pi}{N} n \cdot k}$$



- ⟷ Z-transforms *in limit of infinite lattice*

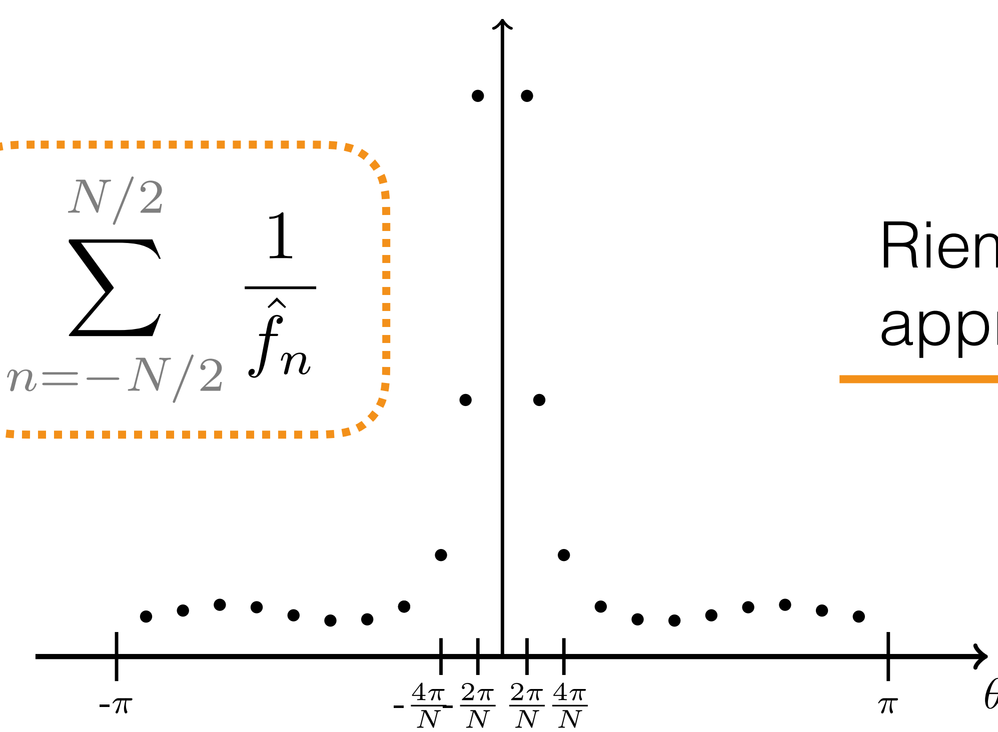


$$\hat{f}_\infty(\theta) := \sum_{k \in \mathbb{Z}^d} f_k e^{-j \theta \cdot k}$$

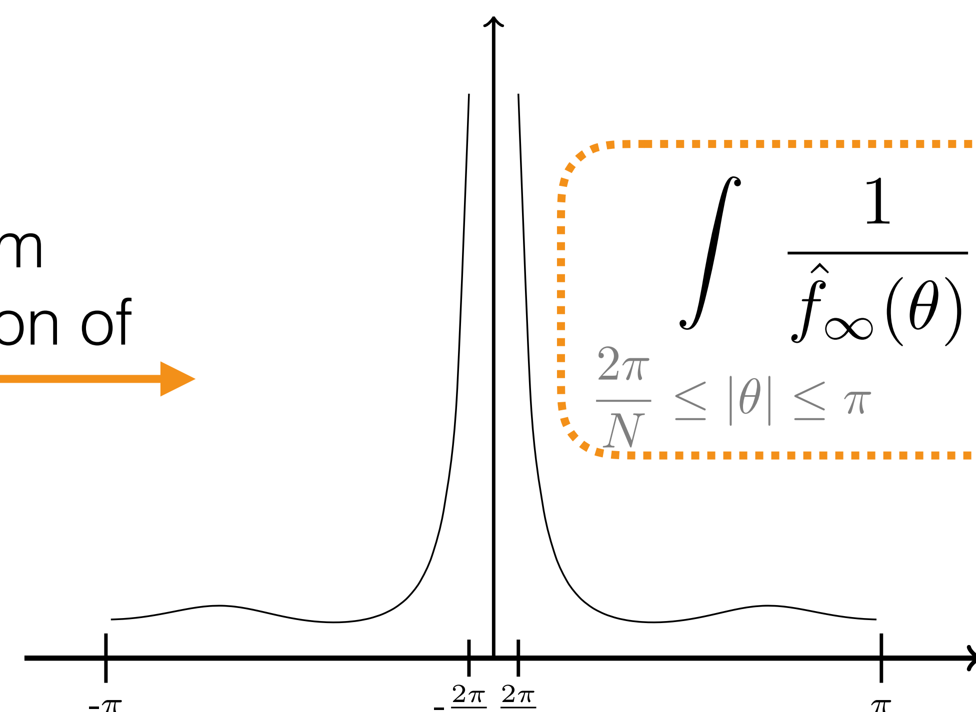


- H_2 norm can be estimated using integral

$$\sum_{n=-N/2}^{N/2} \frac{1}{\hat{f}_n}$$

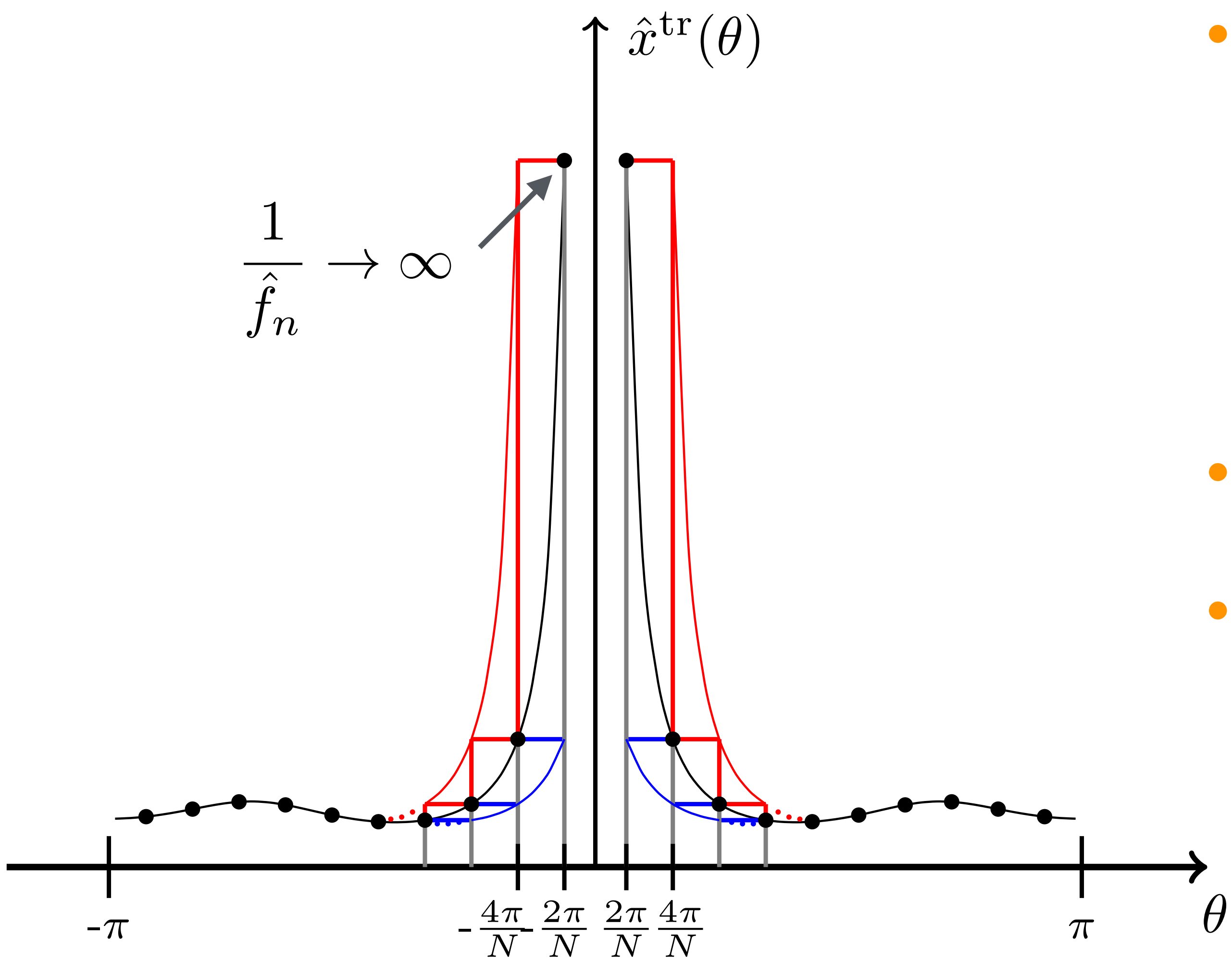


Riemann sum approximation of →



$$\int_{\frac{2\pi}{N} \leq |\theta| \leq \pi} \frac{1}{\hat{f}_\infty(\theta)} \mathrm{d}\theta$$

Asymptotic performance scaling is determined by *how fast* function "blows up"



- Sum estimated through integral like

$$\int_{\frac{2\pi}{N} \le |\theta| \le \pi} \underbrace{\frac{1}{\hat{f}_\infty(\theta)}}_{\hat{x}^{\mathrm{tr}}(\theta)} \mathrm{d}\theta$$

- Typically, singularity at zero
- Order of singularity p determines asymptotic scaling of H_2 norm

Lemma

If $\hat{x}^{\mathrm{tr}}(\theta) \sim \dfrac{1}{|\beta\theta|^p}$, then $V_k \sim \dfrac{1}{\beta^{p/2}} \begin{cases} N^{p-d} & \text{if } d < p \\ \log N & \text{if } d = p \\ 1 & \text{if } d > p \end{cases}$

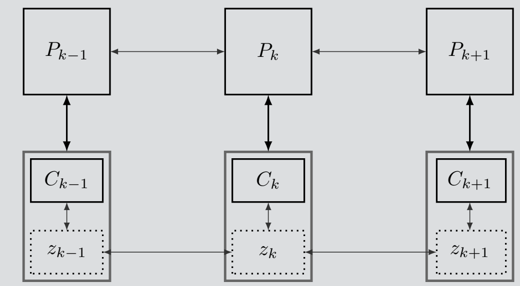
(Notation $u(x) \sim v(x)$ means $u(x)/v(x)$ uniformly bounded)

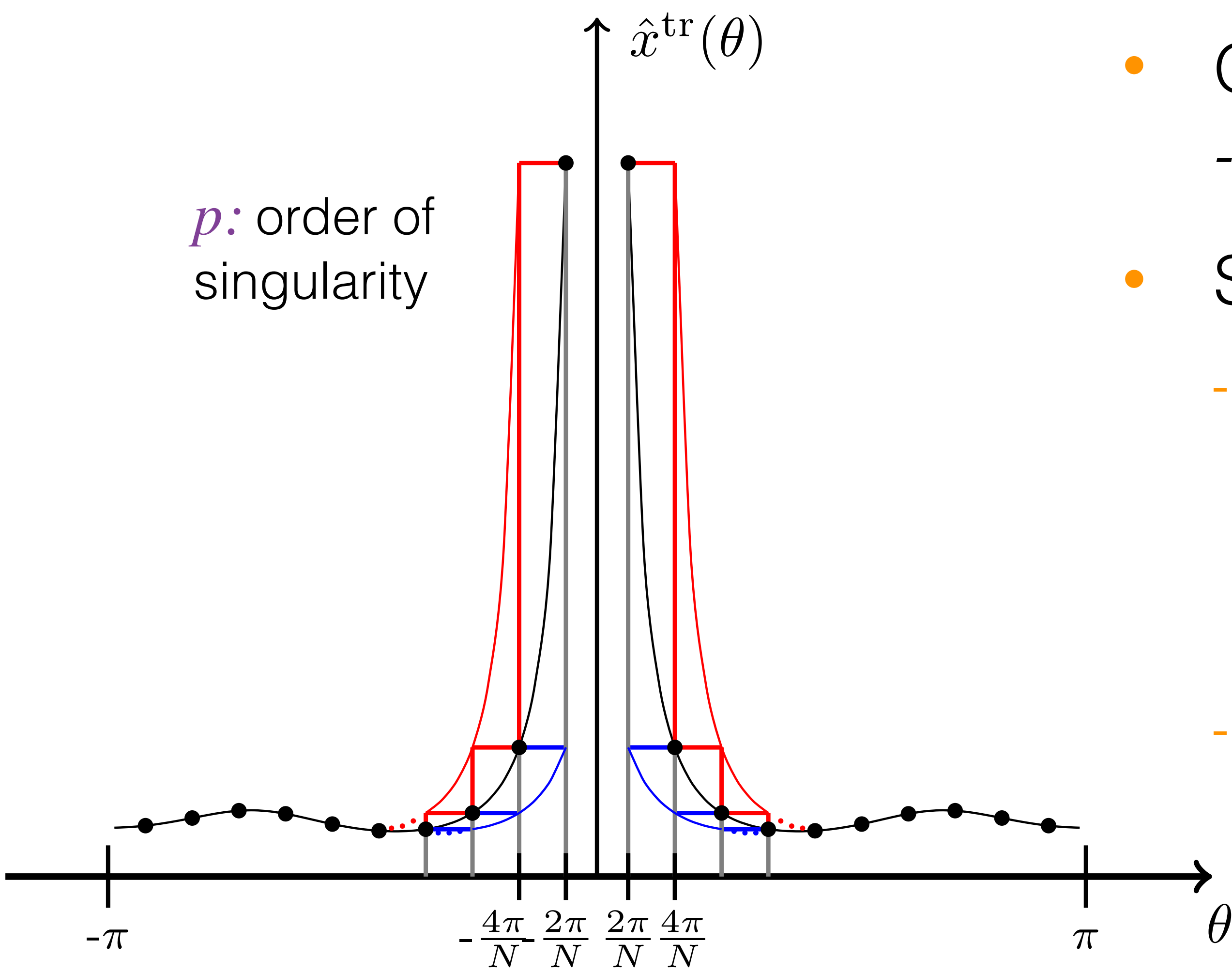
Dynamic feedback does *not* change scaling - if feedback is *relative*

Recall: dynamic feedback laws

$$\begin{bmatrix} \dot{z} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} A & B \\ I & F \end{bmatrix} \begin{bmatrix} z \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w$$

$$\begin{bmatrix} \dot{z} \\ \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} A & B & C \\ 0 & 0 & I \\ I & F & G \end{bmatrix} \begin{bmatrix} z \\ x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} w$$





- Only relative feedback from x, v
  - F, G, B, C all have zero eigenvalues!
- Scaling of integrand unchanged
  - Consensus

$$\hat{x}^{\mathrm{tr}}(\theta) = \frac{-1}{2\hat{f}(\theta) + 2\varphi^c(\theta)} \sim \frac{1}{|\beta\theta|^2} \quad p = 2$$

From dynamic feedback

  - Vehicular formations

$$\hat{x}^{\mathrm{tr}}(\theta) = \frac{d}{2\hat{f}(\theta)\hat{g}(\theta) + 2\varphi^v(\theta)} \sim \frac{1}{|\beta\theta|^4} \quad p = 4$$

From dynamic feedback

Performance improves if absolute feedback available

Asymptotic performance scalings for the vehicular formation problem:

	Static feedback	Dynamic feedback	
Absolute x, v	$V_k \sim \frac{1}{\beta}$		*Same!*
Relative x, absolute v	$V_k \sim \frac{1}{\beta} \begin{cases} M & d=1 \\ \log M & d=2 \\ 1 & d \geq 3, \end{cases}$	$V_k \sim \frac{1}{\beta}$ *Assuming noiseless measurements!*	*Improved performance!*
Relative x, relative v	$V_k \sim \frac{1}{\beta^2} \begin{cases} M^3 & d=1 \\ M & d=2 \\ M^{1/3} & d=3 \\ \log M & d=4 \\ 1 & d \geq 5, \end{cases}$		*Same!*

- Dynamic feedback with absolute velocities substitute absolute positions: *improves performance!*

$$u_k = \text{relative feedback} - g_o v_k + z_k$$

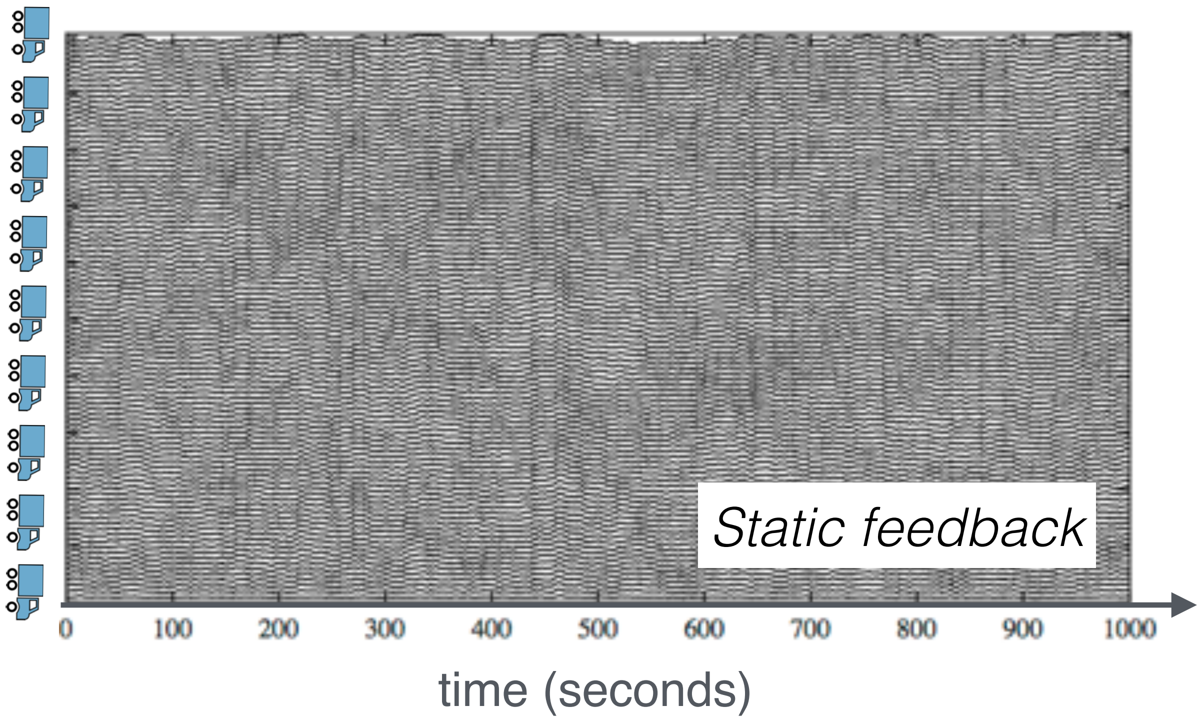
$$\dot{z}_k = a_+(z_{k+1} - z_k) + a_-(z_{k-1} - z_k) - c_o v_k$$

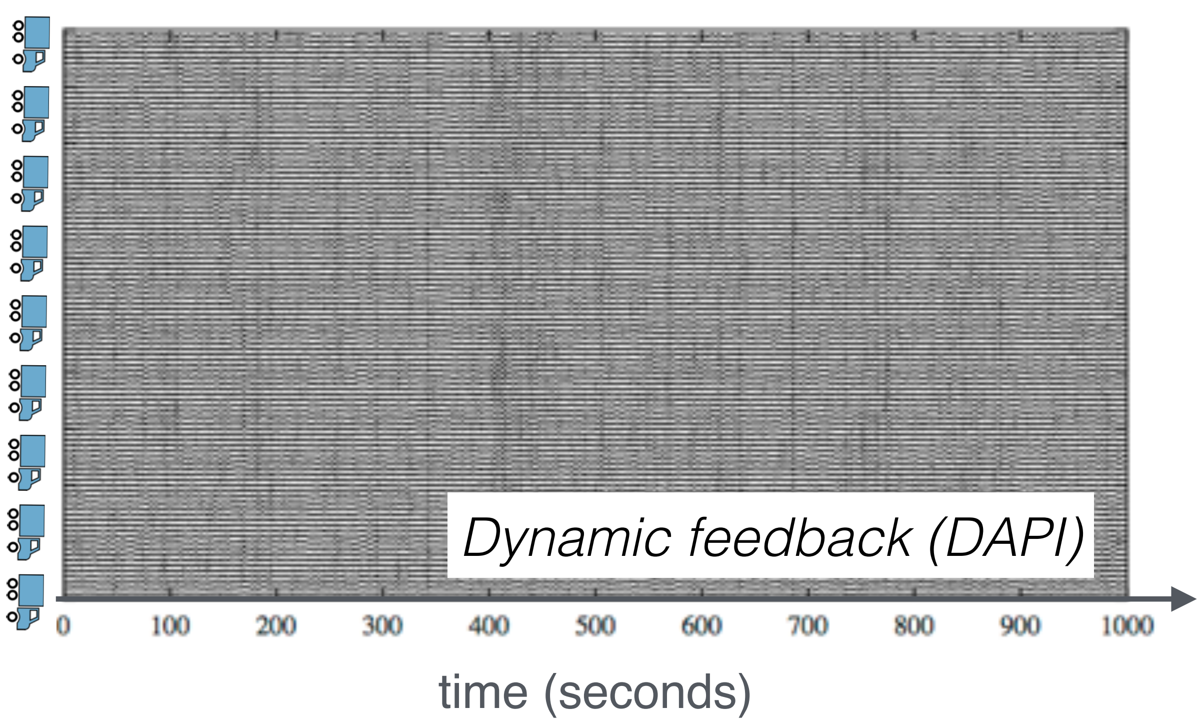
Distributed averaging of z

Distributed averaging PI control (DAPI)

- With noisy measurements - distributed averaging of memory states needed

DAPI improves performance if absolute velocity measurements are available

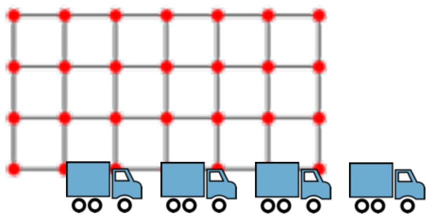




Time trajectories in 100 vehicle platoon w.r.t. leader
(page 51 in thesis!)

- With noise: cannot achieve full coherence
- Still, performance improvement if noise small
- Useful if speedometers available, but absolute position unknown
- Same feedback situation as in power networks (Part 2)!

Summary: Regular lattice networks, coherence



- Objective: Can dynamic feedback improve coherence of consensus and vehicular formation systems?
- Analyze using spatial Fourier transforms, in limit of infinite lattice
- No improvement with only *relative* state feedback
- Distributed PI control improves performance if *absolute* velocity feedback available

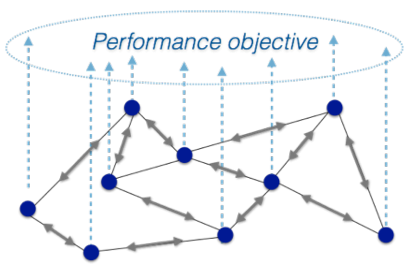
More in the thesis:

- Criteria for stability with dynamic feedback

See also:

E. Tegling, P. Mitra, H. Sandberg, B. Bamieh: Coherence and stability in large-scale networks with distributed dynamic feedback. *In prep.* To be presented at *MTNS*, Minneapolis, Jul 2016.

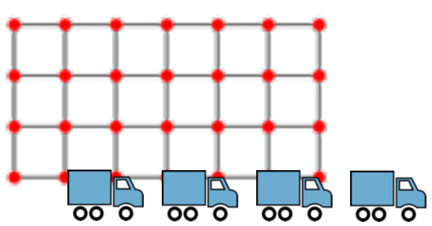
OUTLINE



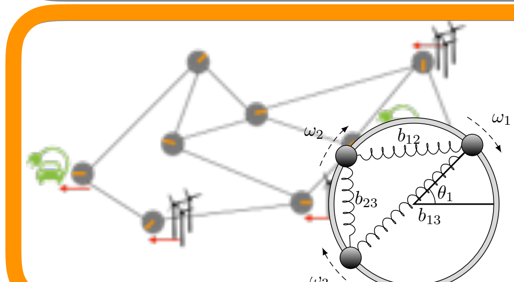
Introduction and problem formulation

$\mathcal{H}_2$

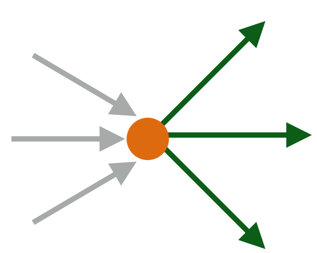
Evaluating input-output performance



Case 1: Regular lattice networks, coherence

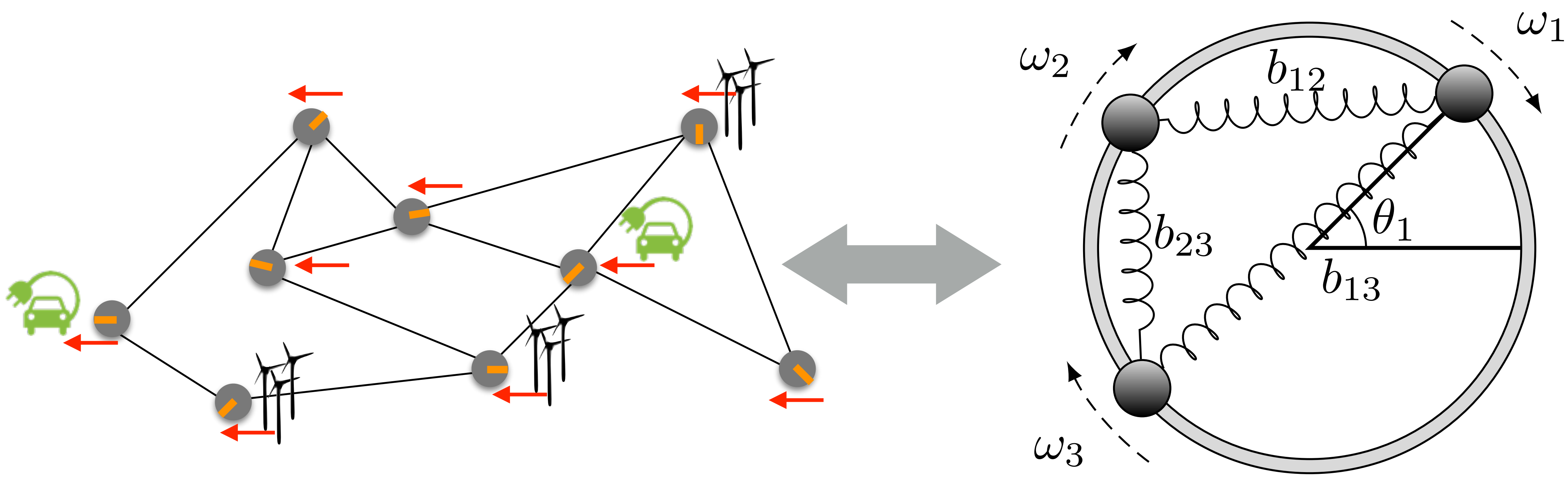


Case 2: Power networks, price of synchrony



Conclusions and future work

Lack of synchrony causes power losses - measure of performance



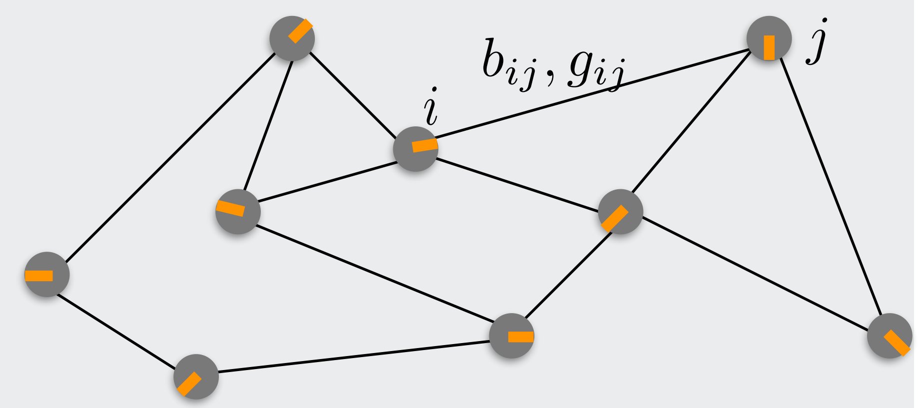
Analogy due to F. Dörfler et al.

Performance measure: *cost* of maintaining synchrony

- Assume stable operating conditions
- Assume distributed stochastic disturbances
- Quantify power losses during re-synchronization
- A *local* performance measure

Power network is modeled through coupled swing equations

- Network: N-node graph representing AC power lines between generators.
- Weighted graph Laplacians L_B, L_G
  - Susceptance matrix L_B, weights b_{ij}
  - Conductance matrix L_G weights g_{ij}



- Swing equation:

$$m_i \ddot{\theta}_i + d\dot{\theta}_i = P_{m,i} - P_{e,i}$$

(θ_i phase angle, m_i inertia, d_i damping)

- Electric power flow: Power injection at node i

$$P_{e,i} = \sum_{j \in \mathcal{N}_i} b_{ij} (\theta_i - \theta_j)$$

($\mathcal{N}_i$ neighbor set of node i, b_{ij} line susceptance)

- System:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}L_B & -M^{-1}D \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} w$$

Power losses measured through appropriate performance output

- Power loss over line i,j (Ohm's law, quadratic approximation):

$$P^{\text{loss}} \approx g_{ij}(\theta_i - \theta_j)^2$$

(g_{ij} line conductance)

- Total losses over network:

$$\mathbf{P}^{\text{loss}} = \sum_{e_{ij} \in \mathcal{E}} g_{ij}(\theta_i - \theta_j)^2 = \theta^* L_G \theta$$

Recall: H_2 norm for general system under white noise input:

$$||H||_2^2 = \lim_{t \to \infty} \mathbb{E}\{y^*(t) y(t)\},$$

- Set performance output to:

$$y(t) = L_G^{1/2} \theta(t)$$

Expected losses now given by the system's H_2 norm

Main result: Losses given by generalized graph Laplacian ratio

- Assume uniform generator damping $d_i = d$

Theorem

$$||H||_2^2 = \frac{1}{2d}\mathrm{tr}\left(L_B^\dagger L_G\right)$$

This represents the expected power loss incurred in maintaining synchrony

- L_B *conductance matrix*
- L_G *susceptance matrix*
- d *generator damping*
- $\dagger$ *Moore-Penrose pseudo inverse*

Special case: no topology dependence if network line ratios are equal

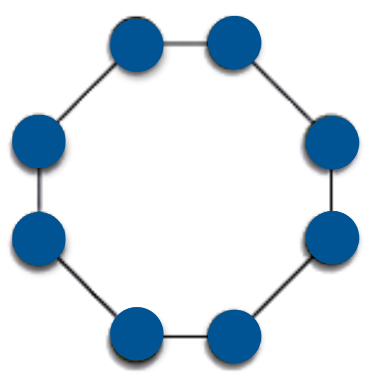
- Assume equal conductance-to-susceptance ratios: $\frac{g_{ij}}{b_{ij}} = \alpha$

Corollary

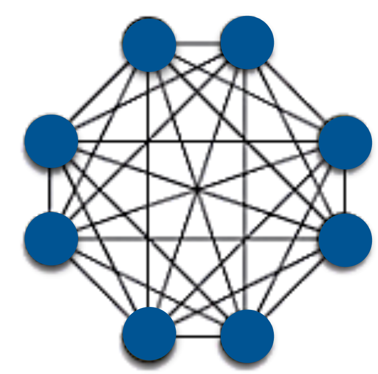
$$||H||_2^2 = \frac{\alpha}{2d}(N-1)$$

N: number of generators

- Grows unboundedly with network size N
- Entirely independent of network topology!

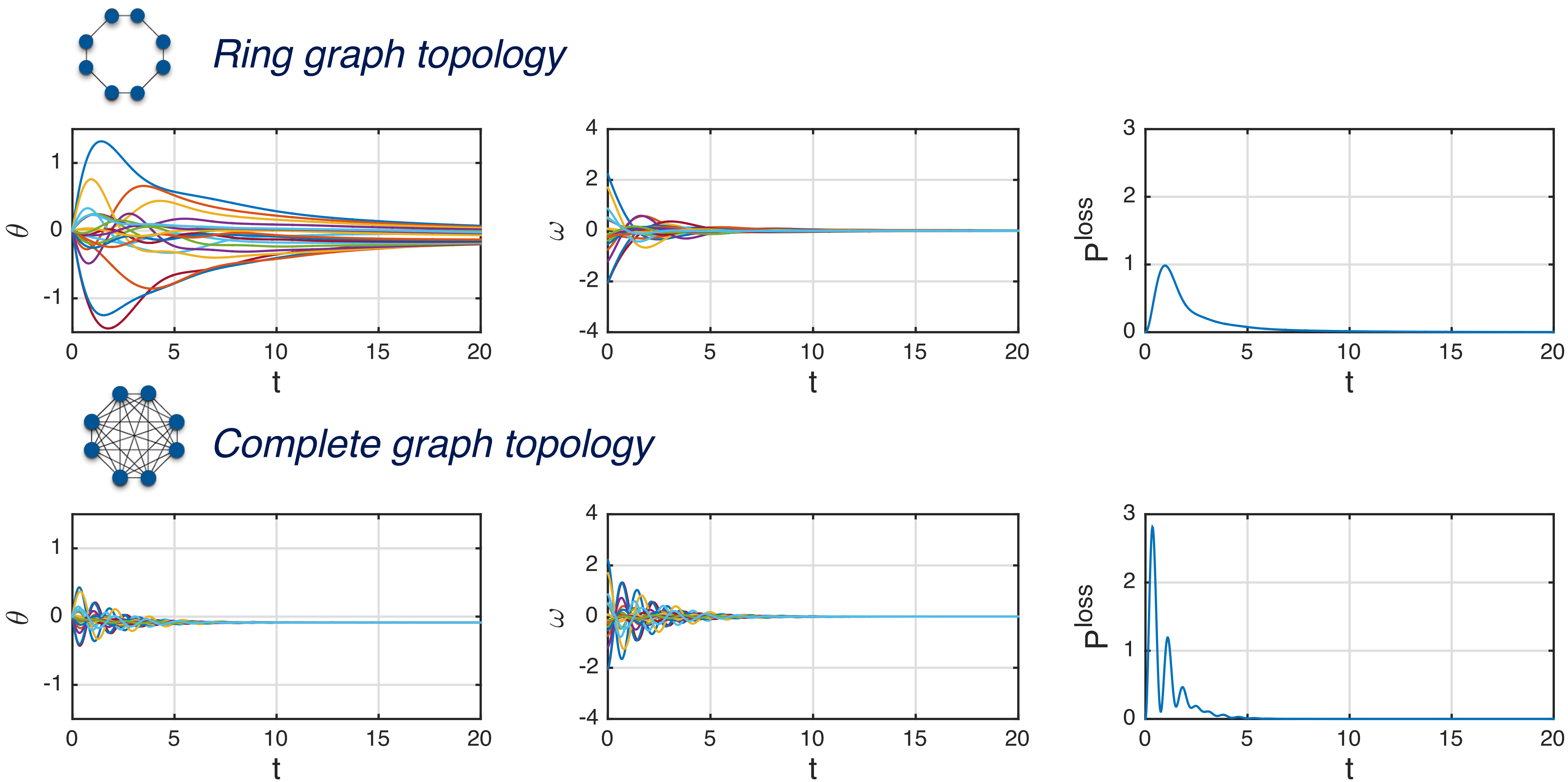


vs.



Less "coherent"	<	More "coherent"
Larger phase fluctuations	>	Smaller phase fluctuations
Less links	<	More links
Same transient power losses	=	Same transient power losses

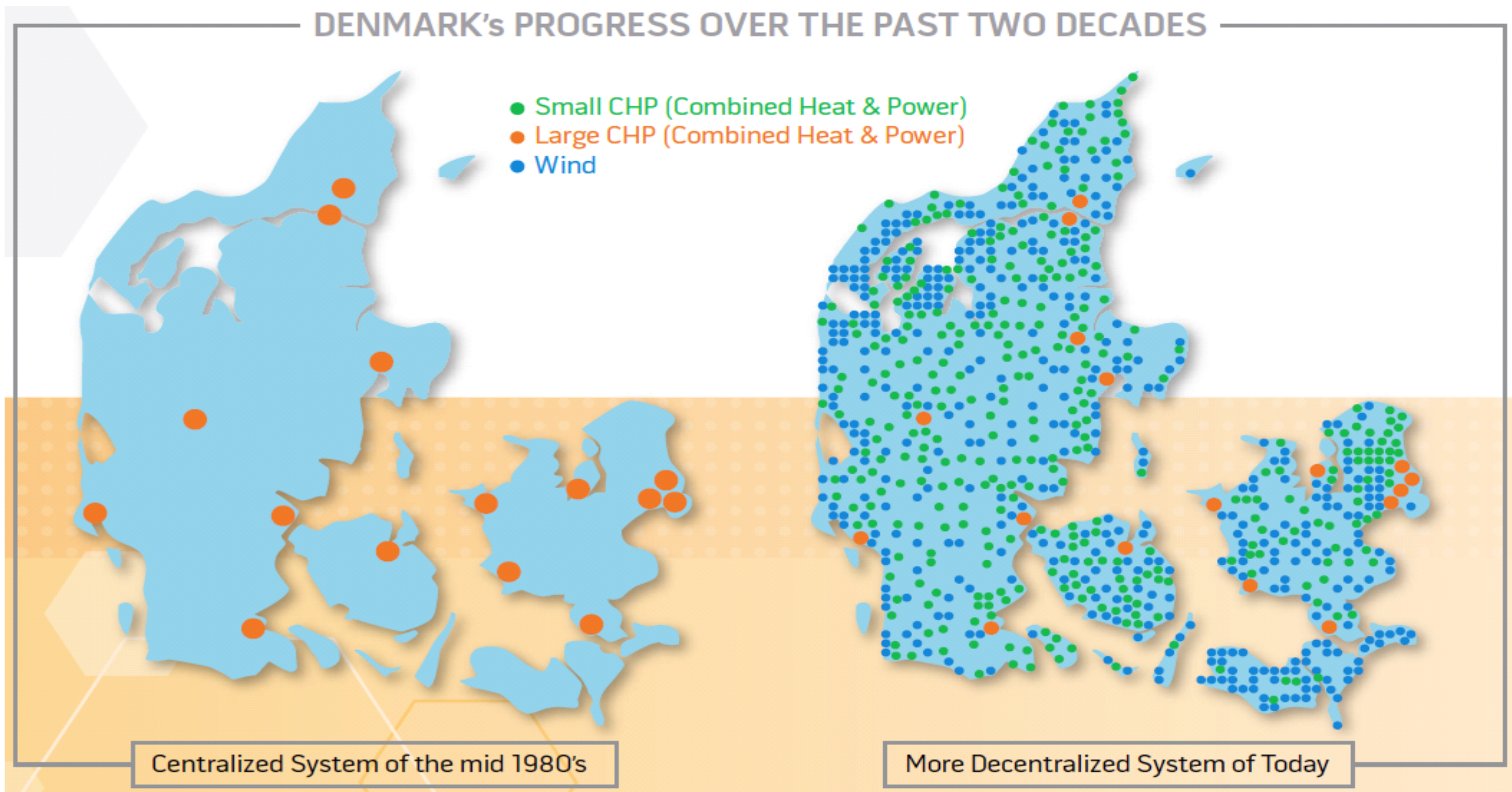
High connectivity gives faster synchronization, but requires more power flows



Synchronization transients in 20 node networks

- Faster synchronization and tighter phases in complete graph
- Same total losses over the transient

Implications: potential for large losses with distributed generation

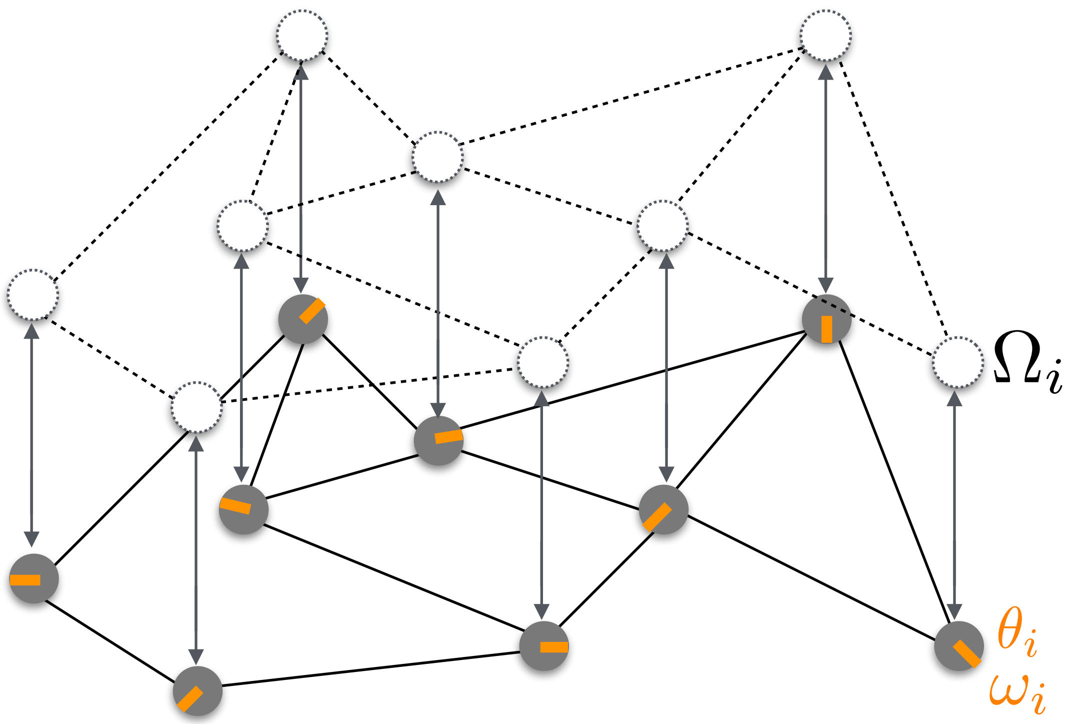


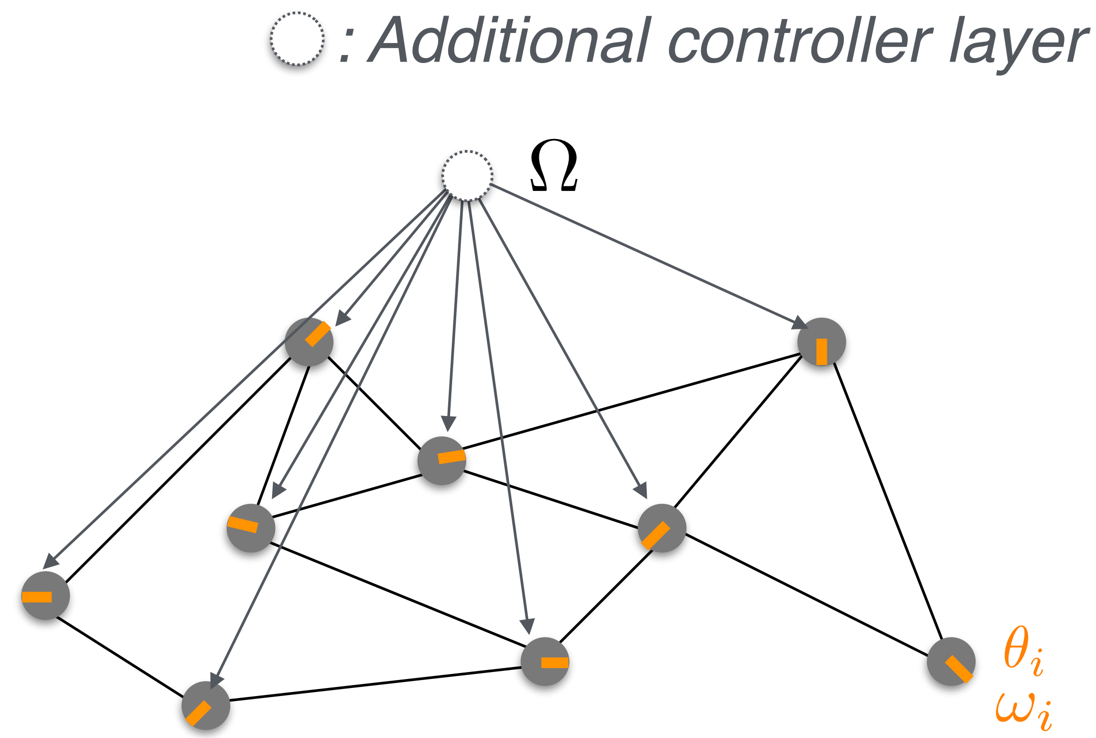
Source: DoE, Smart Grid Intro, 2008

- Transient losses scale with network size
- No improvement by increasing number of links
- Fundamental limitation - if using *electric power flows* for synchronization

Can dynamic feedback improve performance?

Can dynamic feedback (PI control) improve performance?





Distributed averaging PI control (DAPI)

$$\dot{\omega}_i = [\text{swing equation}] + \Omega_i$$

$$q\dot{\Omega}_i = -\omega_i - \sum_{j \in \mathcal{N}_i^C} c_{ij}(\Omega_i - \Omega_j)$$

Distr. averaging of Ω

Centralized averaging PI control (CAPI)

$$\dot{\omega}_i = [\text{swing equation}\ ] + \Omega$$

$$q\dot{\Omega} = -\frac{1}{N}\sum_{i \in \mathcal{V}} \omega_i$$

Centr. averaging of ω

($\mathcal{N}_i^C$ *neighbor set of node* i, c_{ij} *constant gains, here* $c_{ij} = \gamma\, b_{ij}$, q *integral gain)*

- Controllers proposed by Simpson-Porco *et al.* (2013), and Andreasson *et al.* (2014) for elimination of stationary control errors

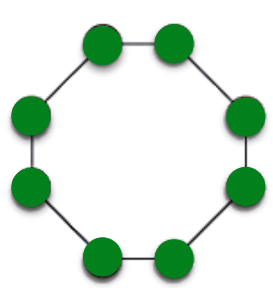
CAPI leaves performance unchanged, while DAPI reduces losses

Theorem

$$||H_{\mathrm{CAPI}}||_2^2 = \frac{\alpha}{2d}(N-1) \;>\; ||H_{\mathrm{DAPI}}||_2^2 = \frac{\alpha}{2d}\sum_{n=2}^{N} \frac{1}{1+\frac{\gamma\tau\lambda_n+q}{\gamma\lambda_n(\gamma\tau\lambda_n+q)+q^2 m\lambda_n}}$$

N-1 terms, each<1

Same as before! *Topology dependent!*



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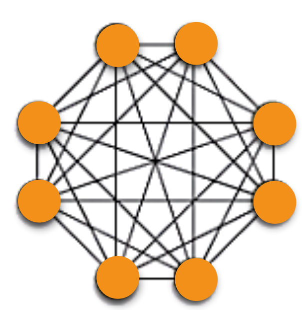
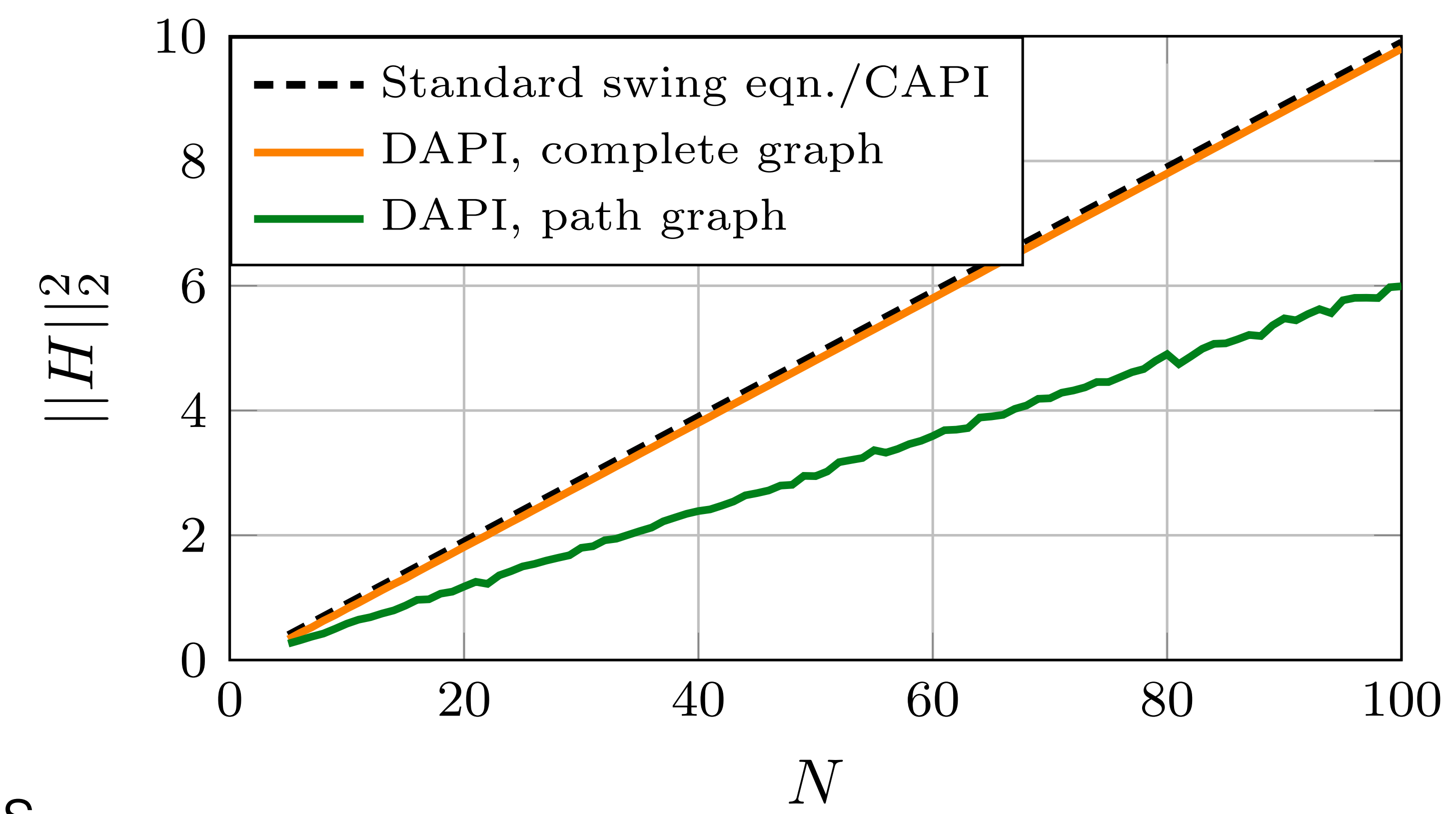
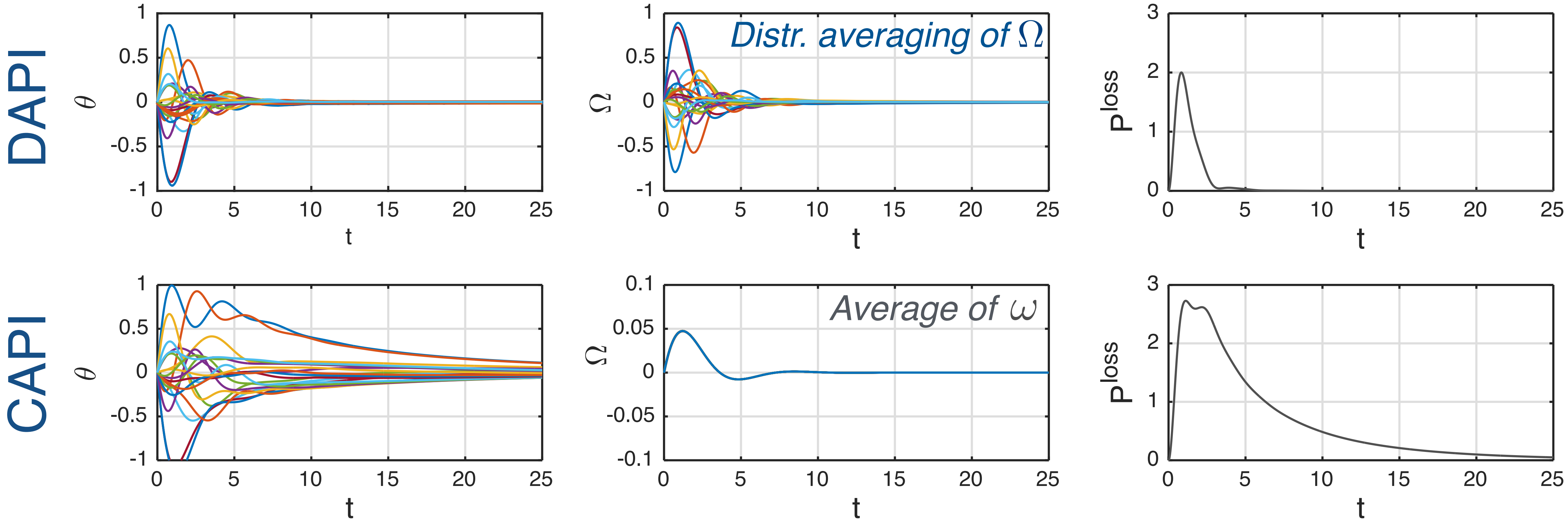




- DAPI control reduces losses
- Smaller losses for sparse topologies
- Losses still grow with network size N

Self-damping key to improved performance



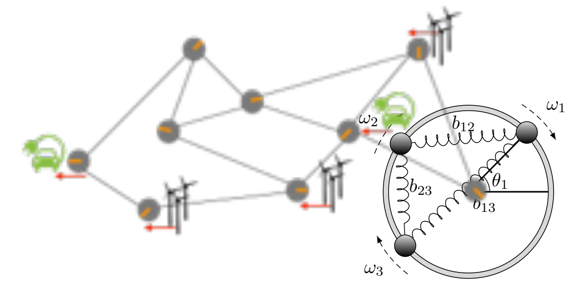
Synchronization transients in 20 node network

- DAPI provides substitute for absolute phase feedback (self-damping)

$$\dot{\omega} = \text{relative feedback} - d_i\omega_i - \underbrace{\frac{1}{q}\int_0^t \omega(\tau)d\tau}_{\to\theta_i}$$

- "Cheaper" to rely on self-damping as power flows associated with costs
- Need to align with neighbors — but too strong alignment reduces self-damping effect

Summary: Power networks, price of synchrony

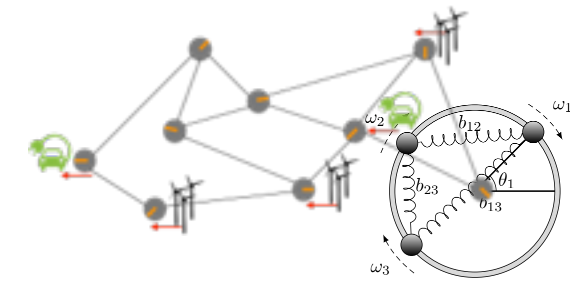


- Measure performance of power networks in terms of losses incurred in synchronization
- With standard control, losses increase with network size, but do not depend on network connectivity
- Distributed PI control can reduce losses by emulating self-damping

More in the thesis

- Elaboration on H_2 norm interpretation in terms of power losses
- Renewable energy integrated grids
  - microgrids with variable voltages
  - heterogeneous oscillator networks
- Optimal configuration of the DAPI controller

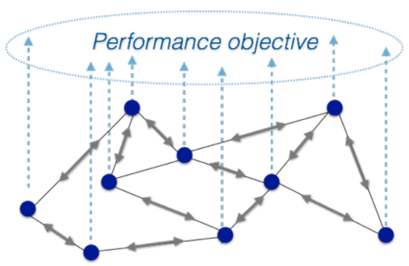
Summary: Power networks, price of synchrony



See also:

- E. Tegling, B. Bamieh and D. F. Gayme: The price of synchrony: Evaluating the resistive losses in synchronizing power networks. *IEEE TCNS*, Sep 2015
- E. Sjödin and D.F. Gayme: Transient losses in synchronizing renewable energy integrated power networks. *ACC*, Jun 2014.
- E. Tegling, D. F. Gayme, and H. Sandberg: Performance metrics for droop-controlled microgrids with variable voltage dynamics. *CDC*, Dec 2015.
- E. Tegling, M. Andreasson, J. W. Simpson-Porco, and H. Sandberg: Improving performance of droop-controlled microgrids through distributed PI-control. *ACC*, Jul 2016.

OUTLINE



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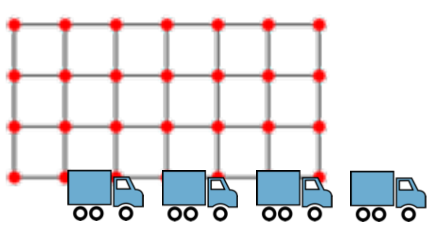
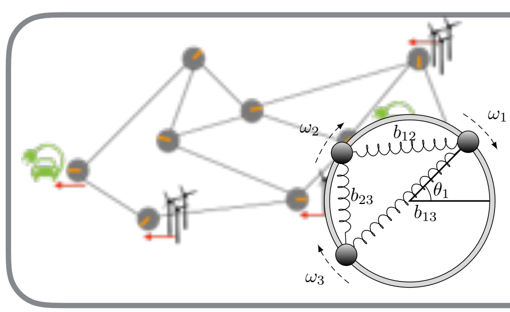
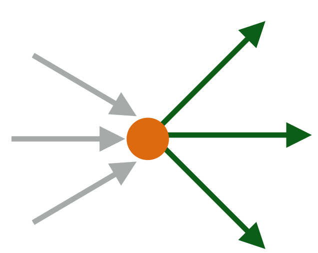


Case 1: Regular lattice networks, coherence



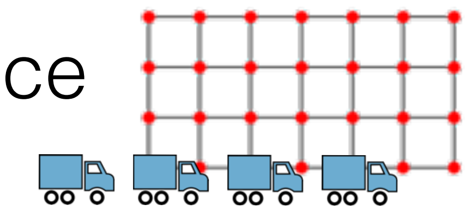
Case 2: Power networks, price of synchrony



Conclusions and future work

Scaling of performance is a limitation in networked systems with local feedback

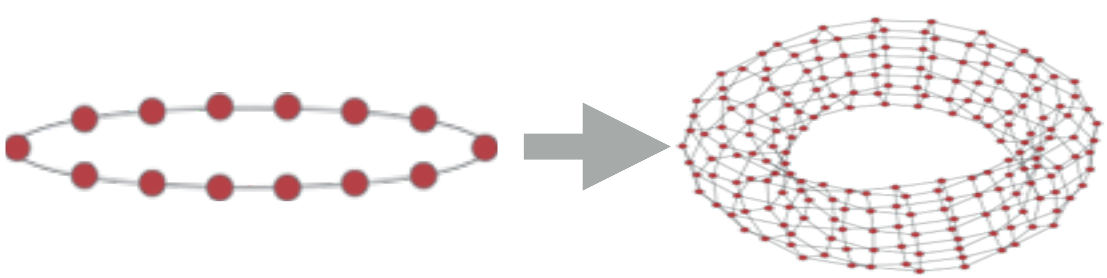
Case 1: Coherence



- *Global* disorder
- Measured per node

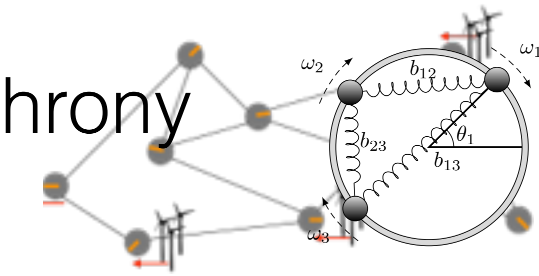
- Scaling (worst case):

$$V_k \sim N^3$$



- Performance improvement

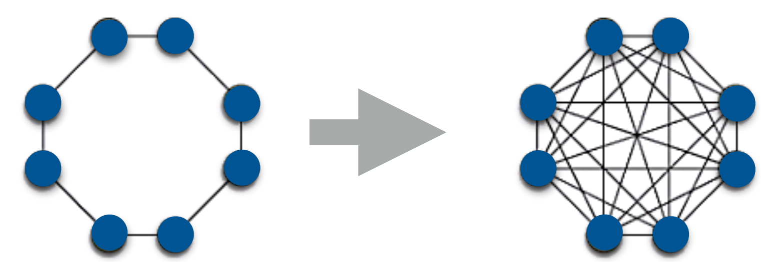
Case 2: Price of Synchrony



- Power losses caused by *local* disorder
- Measured over entire network

- Scaling:

$$\mathbf{P}^{\text{loss}} \sim N$$



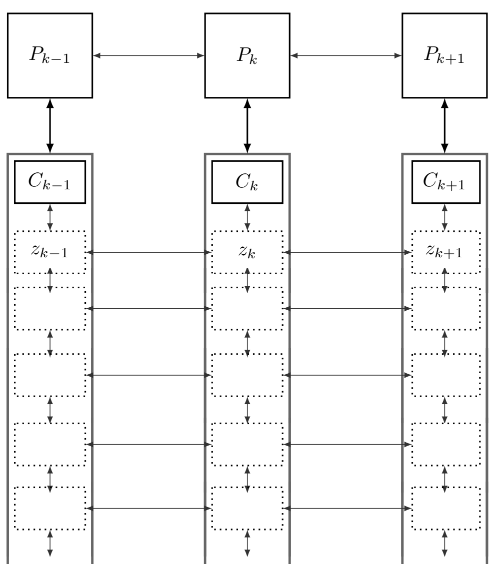
- *No* performance improvement

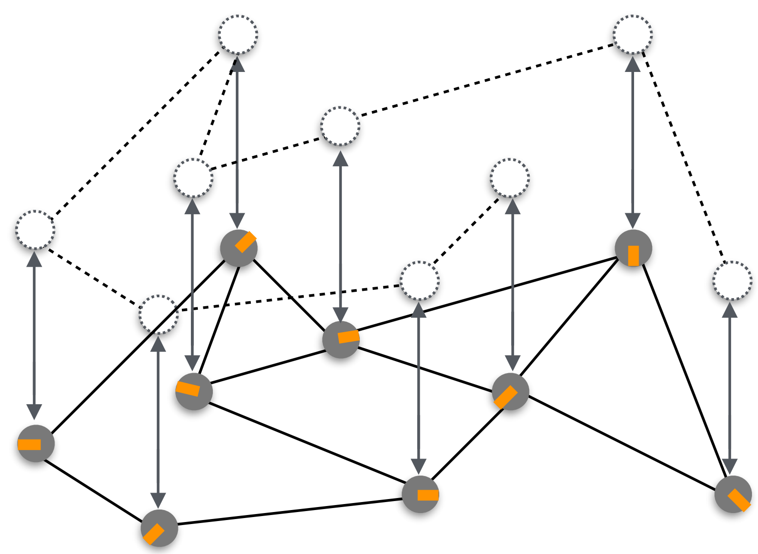
- Absolute feedback key in improving performance
- Dynamic control laws can emulate absolute feedback, if *distributed*!
- Limitation in terms of scaling remains

Future work includes further exploration of distributed dynamic feedback

<u>Topics to explore</u>

- Higher-order controllers
- DAPI control architecture
- Other performance metrics





$\mathcal{H}_2$ vs. $\mathcal{H}_\infty$ vs. ...

Thank you!