On the Coherence of Large-Scale Networks with Distributed PI and PD Control

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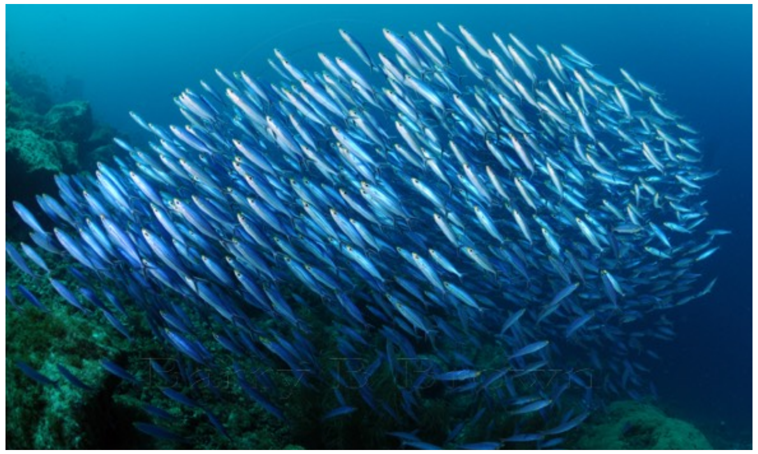
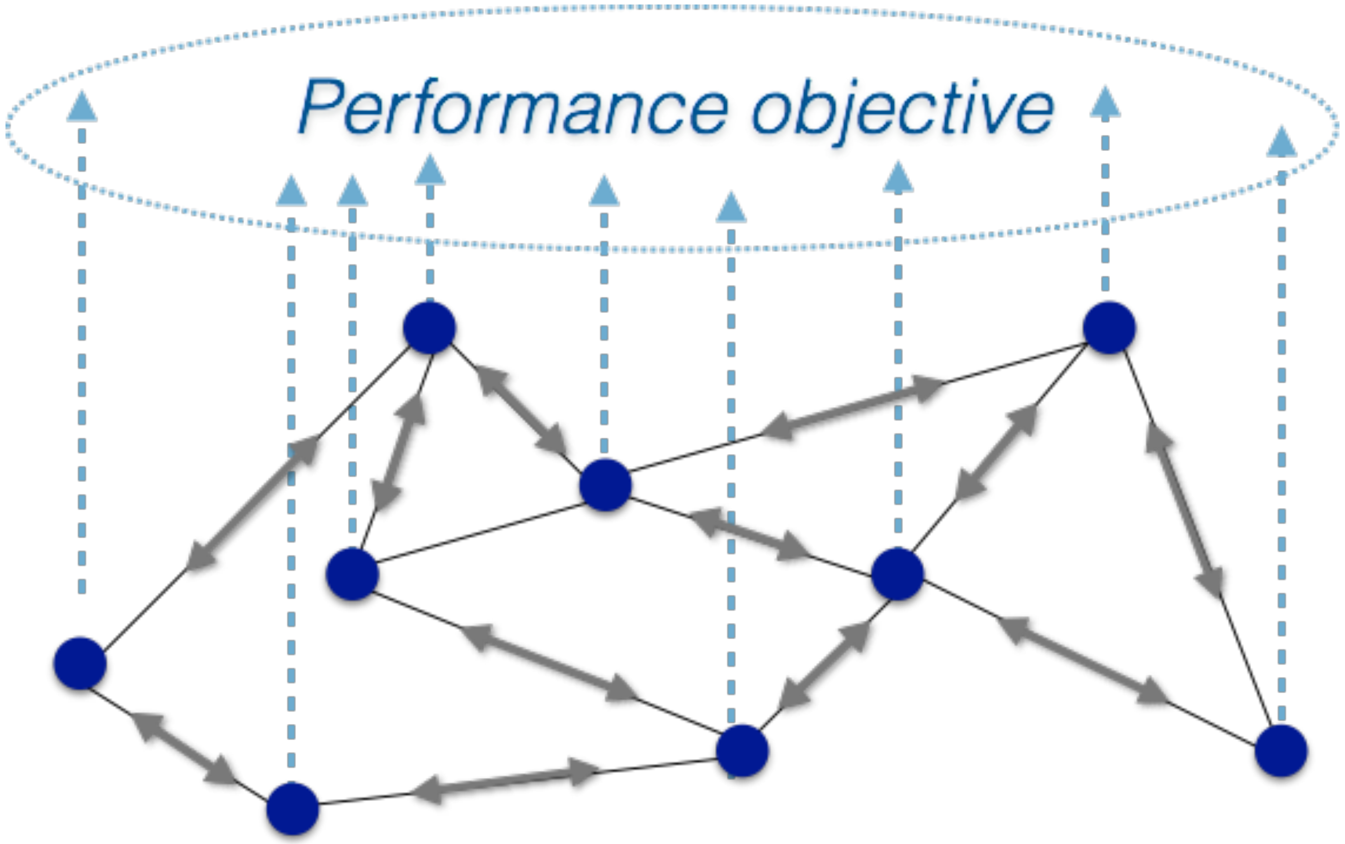
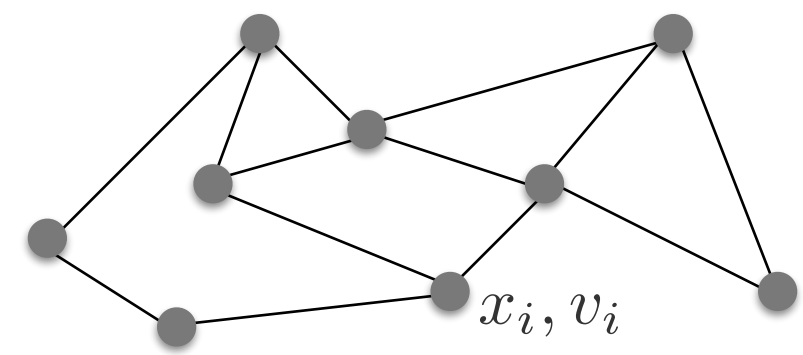
Networked systems: *global* objectives, but *local* feedback





Are there limitations to network *performance*?

Problem setup: Linear, second-order consensus subject to distributed disturbances



- Consider connected graph with N agents
- Each agent i is double-integrator

$$\dot{x}_i(t) = v_i(t)$$
$$\dot{v}_i(t) = u_i(t) + w_i(t)$$

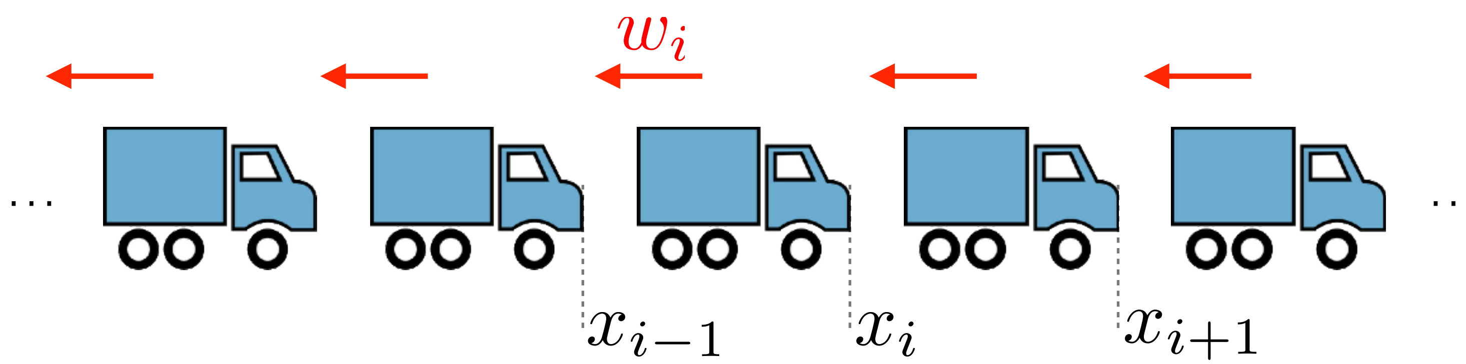
- Control objective: follow trajectory $\bar{x}_i(t) := \bar{v}t + \delta_i$
- Standard linear consensus / *Proportional (P) control*

$$u_i = \underbrace{-\sum_{j \in \mathcal{N}_i} f_{ij}(x_i - x_j) - \sum_{j \in \mathcal{N}_i} g_{ij}(v_i - v_j)}_{\text{Relative feedback}} \; \cancel{\underbrace{- f_0 x_i - g_0 v_i}_{\text{Absolute feedback}}}$$

- Let each agent be subject to stochastic disturbance $w_i(t)$

x_i, v_i *deviation from state trajectory,* δ_i *setpoint,* f_{ij}, g_{ij}, f_0, g_0 *fixed, constant gains,* $\mathcal{N}_i$ *neighbor set*

Example 1: Large-scale vehicle platoons



- Objective: follow trajectory $\bar{x}_i(t) := \bar{v}t + \delta_i$
  - common cruising speed $\bar{v}$
  - tight constant spacing Δ, so that $\delta_i = \Delta i$

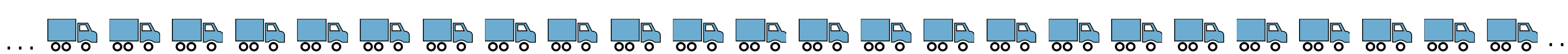
- Example control law: look-ahead, look-behind control

$$u_i = f_+(x_{i+1} - x_i) + f_-(x_{i-1} - x_i) + g_+(v_{i+1} - v_i) + g_-(v_{i-1} - v_i)$$

(f_+, f_-, g_+, g_- constant gains)

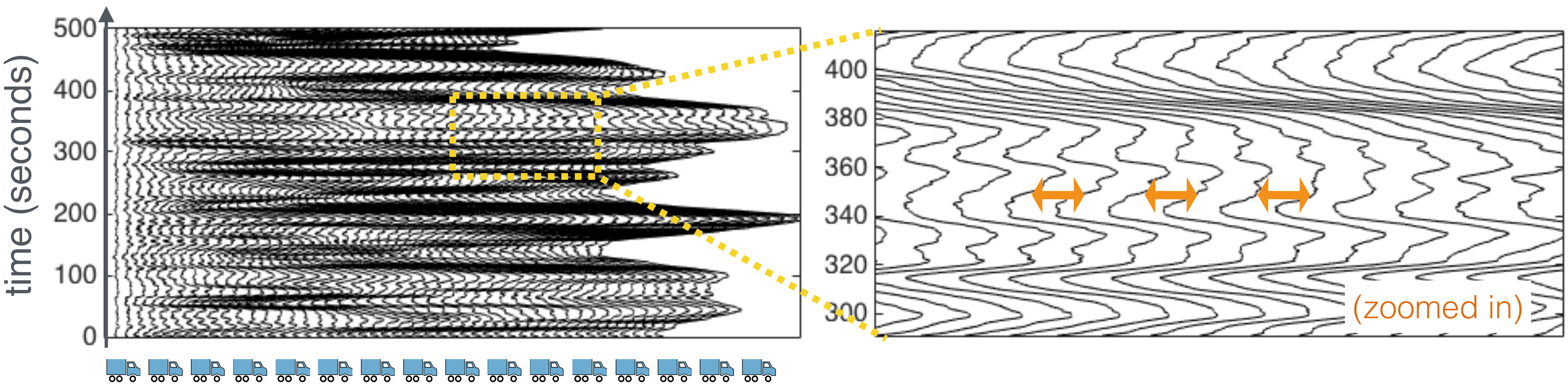
- With disturbances: objectives only achieved approximately

- What happens if the platoon grows?



Example 1 (contd.): Performance issues if control based on relative feedback

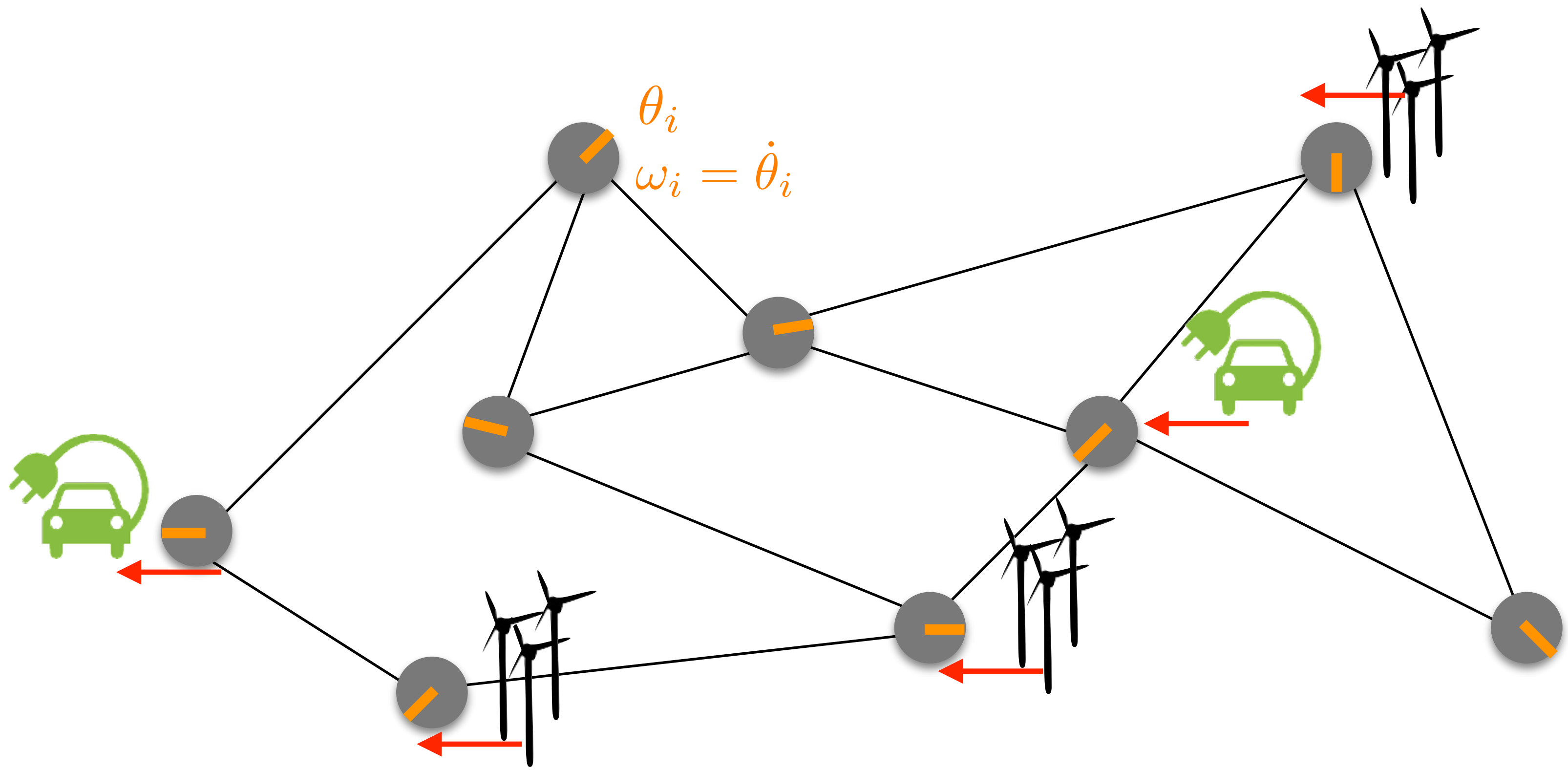
Time trajectories of 100 vehicles, relative to leader, seen from above



- Formation is stable
- Spacings ⟷ are well-regulated (no collisions!)
- However - not a *rigid* formation, not *coherent!*
- Fundamental limitation to local, static feedback (Bamieh *et al.*, 2012)

Can dynamic feedback (PID control) help?

Example 2: Frequency control in power networks



- Objectives:
  - common, steady frequency $\bar{\omega}$ (60 Hz)
  - phase angles at equilibrium $(\theta_i - \theta_j) \sim P_i^*$

- Swing equation, or "droop control" (linearized)

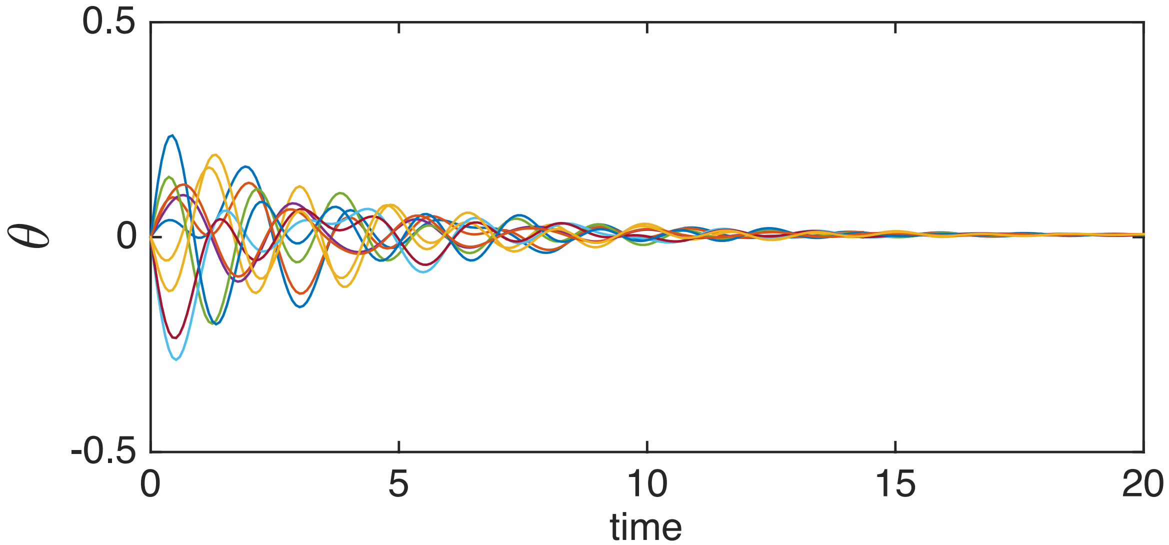
$$m_i \dot{\omega}_i = -d_i \omega_i - \sum_{j \in \mathcal{N}_i} b_{ij}(\theta_i - \theta_j) + w_i$$

(b_{ij} line susceptance, m_i inertia, d_i damping)

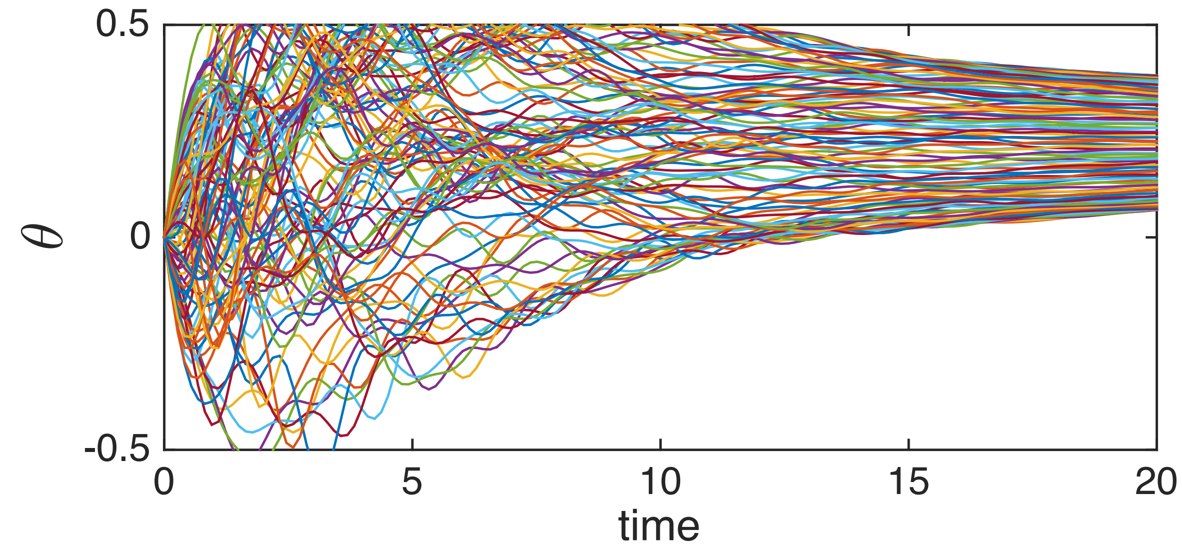
- Transition to distributed generation affects power system dynamics
  - More disturbances, (many) more generators

Example 2 (contd.): Issues with scalability of standard droop controller

- Simulation of droop control on 10 vs 100 node network (tree graph)

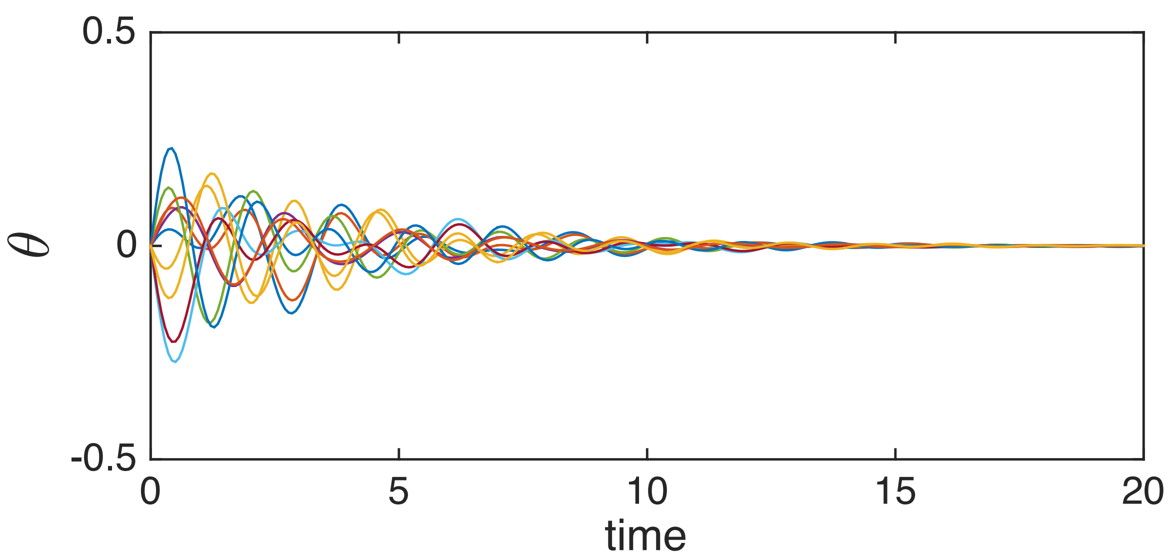


$N = 10$

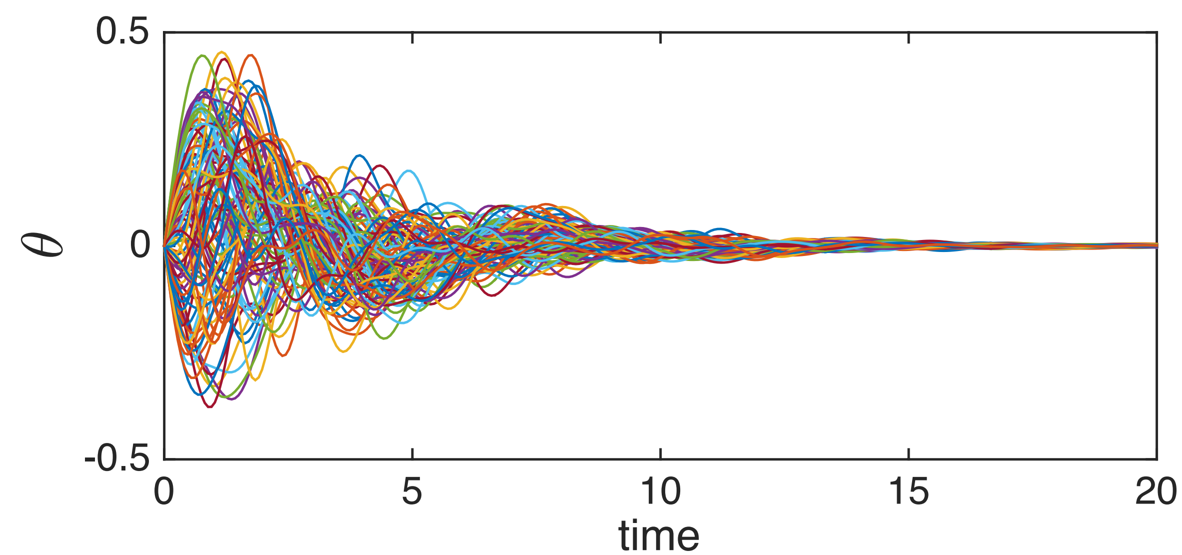


$N = 100$

- **Today:** Better scalability with distributed PI-control (dynamic feedback)



$N = 10$



$N = 100$

Problem setup: Performance is quantified through a measure of network coherence

- Consider each agent's deviation from the network average

$$y_i^{\mathrm{dav}} = x_i - \frac{1}{N}\sum_{j=1}^{N} x_j$$

- Characterizes rigidity, coherence

- Performance is measured as *variance* of performance output, normalized by *N*

$$V_N = \frac{1}{N}\mathbb{E}\{y^T(t)y(t)\}$$

- Interested in the *scaling* of the output variance with network size

Summary: We characterize scalability of distributed control laws

- **Model:** Second order consensus with performance output

$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} = \begin{bmatrix} 0 & I \\ -\mathcal{L}_F - f_0 I & -\mathcal{L}_G - g_0 I \end{bmatrix}\begin{bmatrix} x \\ v\end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w$$

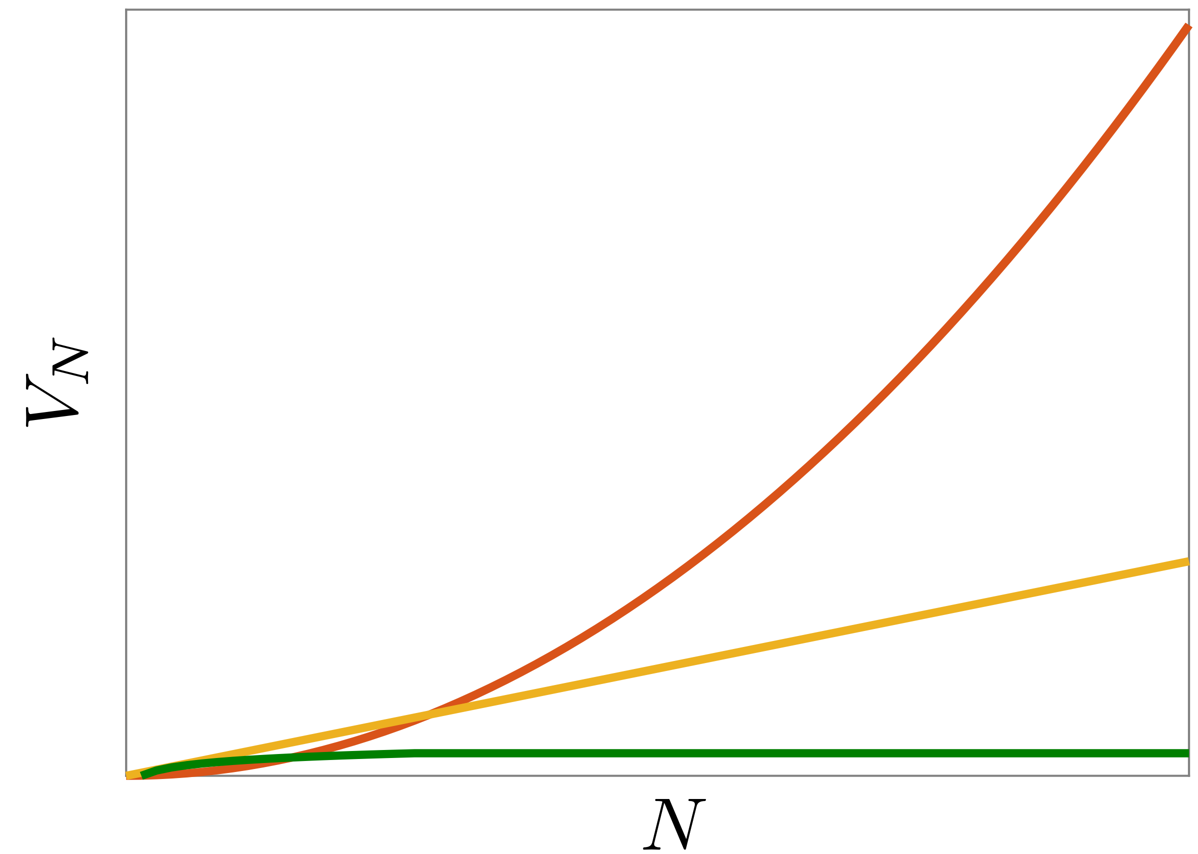
$$y = \begin{bmatrix} I - \frac{1}{N}\mathbf{1}\mathbf{1}^T & 0 \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix}$$

($\mathcal{L}_F, \mathcal{L}_G$ weighted graph Laplacians, assume $\mathcal{L}_F = f\mathcal{L}, \mathcal{L}_G = g\mathcal{L}$ for some (weighted) $\mathcal{L}$)

  - Absolute feedback from x (v) if f_0 (g_0) nonzero

- **Performance evaluation:**
  - Consider (asymptotic) scaling of variance $V_N = \frac{1}{N}\mathbb{E}\{y^T(t)y(t)\}$
  - Control law scales well only if V_N *bounded* in N

- **Objective:** Compare static vs. dynamic feedback



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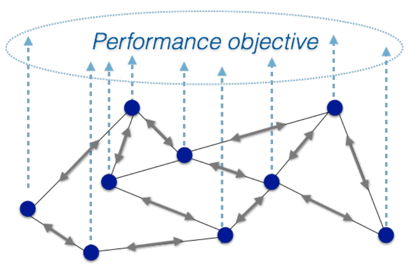
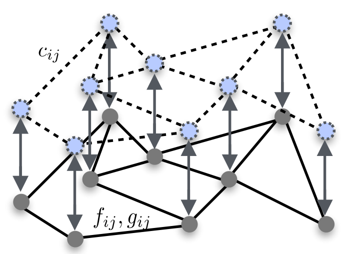


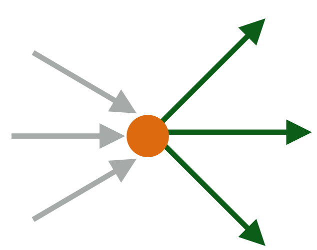
Introduction and problem formulation

$\mathcal{H}_2$

Evaluating input-output performance



Distributed PI and PD control



Conclusions and future work

Performance is evaluated through input-output H_2 norms

Consider general linear system under white noise input

$$H: \quad \begin{aligned} \dot{x} &= \mathcal{A}x + \mathcal{B}w \\ y &= \mathcal{C}x \end{aligned} \qquad (1)$$

Recall:

Need to evaluate $V_N = \frac{1}{N}\mathbb{E}\{y^T y\}$, with $y = \left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) x$

Lemma:

The squared H_2 norm of (1) from input w to output y gives

$$||H||_2^2 = \lim_{t\to\infty} \mathbb{E}\{y^T(t)y(t)\},$$

That is, the steady state output variance.

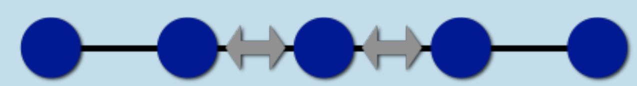
Evaluating system performance amounts to evaluating H_2 norms!

Eigenvalues near zero cause bad performance

Theorem

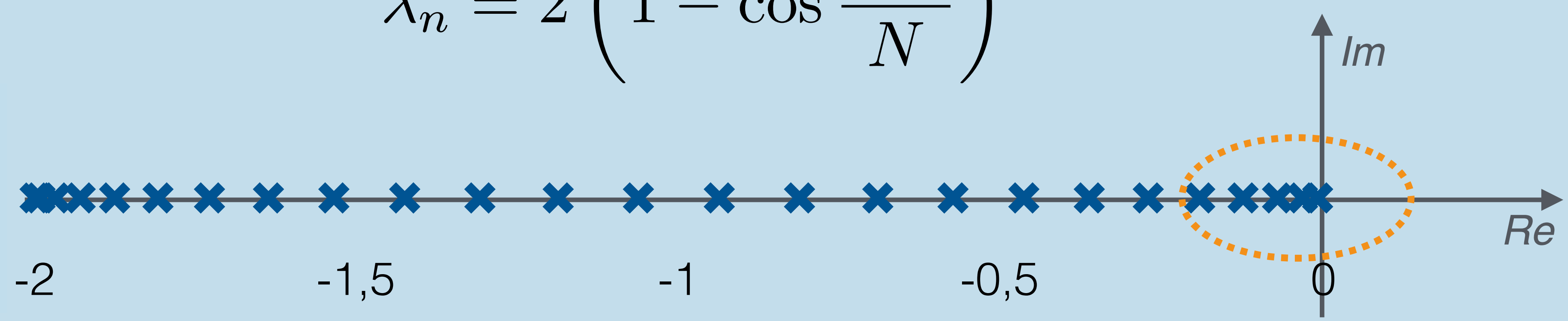
$$V_N = \frac{1}{N}||H||_2^2 = \frac{1}{2N}\sum_{n=1}^{N-1} \frac{1}{(f_0 + f\lambda_n)(g_0 + g\lambda_n)}$$

<u>*Example (Ring graph, uniform weights):*</u>



- Eigenvalues

$$\lambda_n = 2\left(1 - \cos\frac{2\pi n}{N}\right)$$



- As N grows: Arbitrarily many λ_n increasingly close to zero
- Sum blows up, unless $f_0, g_0 \neq 0$, i.e., absolute feedback

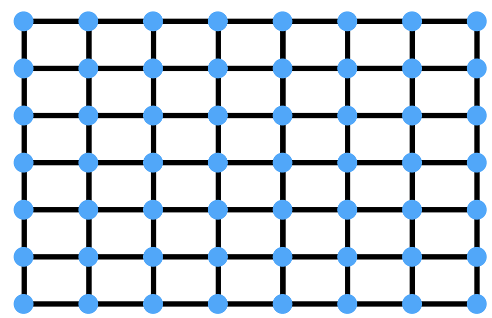
Precise scaling of V_N in N can be determined for regular graphs

P-control scales badly in sparse networks, unless absolute feedback available

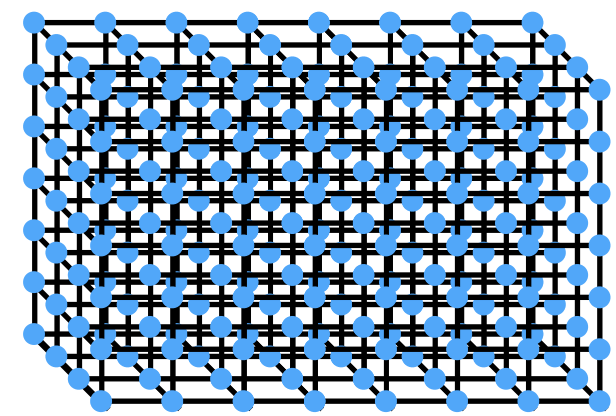
- *Recall*: $u_i = \underbrace{-\sum_{j\in\mathcal{N}_i} f_{ij}(x_i - x_j) - \sum_{j\in\mathcal{N}_i} g_{ij}(v_i - v_j)}_{\text{Relative feedback}} \underbrace{- f_0 x_i - g_0 v_i}_{\text{Absolute feedback}}$
- Let network be d-dimensional lattice

$d = 1$

$d = 2$



$d = 3$



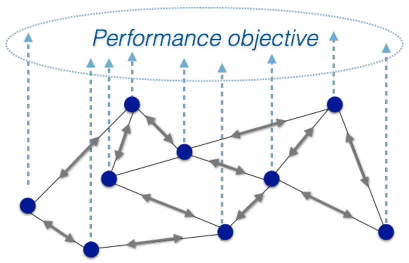
$\cdots$

<u>Asymptotic performance scalings with static feedback (see e.g. Bamieh *et al.*, 2012)</u>

Up to a constant independent of gain parameter β *and network size* N

Relative x, relative v	Relative x, absolute v, Absolute x, relative v	Absolute x, absolute v
$V_N \sim \frac{1}{\beta^2}\begin{cases} N^3 & d=1 \\ N & d=2 \\ N^{1/3} & d=3 \\ \log N & d=4 \\ 1 & d\geq 5 \end{cases}$	$V_N \sim \frac{1}{\beta}\begin{cases} N & d=1 \\ \log N & d=2 \\ 1 & d\geq 3 \end{cases}$	$V_N \sim \frac{1}{\beta}$

OUTLINE



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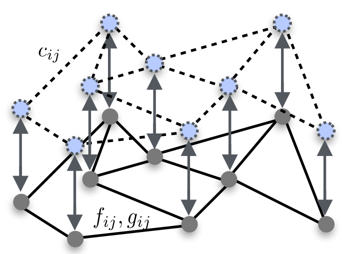
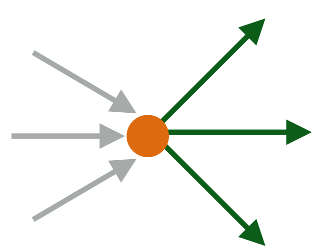


Distributed PI and PD control



Conclusions and future work

Various strategies proposed to deal with performance limitations

- Assign select leaders with absolute measurement (1^{st} order consensus)
  - S. Patterson et al. “Leader selection for optimal network coherence,” CDC 2010
  - F. Lin et al. “Algorithms for leader selection in stochastically forced consensus networks,” TAC 2014
  - M. Pirani et al. “Coherence and convergence rate in networked dynamical systems,” CDC 2015
- Optimize gains, change symmetries
  - T. Summers et al. “Topology design for optimal network coherence,” ECC 2015
  - F. Lin et al. “Optimal control of vehicular formations with nearest neighbor interactions, TAC 2012
- *Here*: use distributed PID-control
  - M. Andreasson et al. “Distributed control of networked dynamical systems: Static feedback, integral action and consensus,” TAC 2014
  - D. Lombana and M. di Bernardo, “Distributed PID control for consensus of homogeneous and heterogeneous networks, TCNS 2016

Idea: use derivative or integral action to substitute unavailable measurement

Derivative action

Absolute x-measurement

- Derivative of x-measurement corresponds to v

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = v_i(t)$$

- *Ideally:* same performance as with absolute feedback in x,v
- Ideal derivative action not possible to implement + sensitive to noise

Integral action

Absolute v-measurement

- Integral of v-measurement corresponds to x

$$\int_0^t v_i(\tau)d\tau = x_i(t) - x_i(0)$$

- *Ideally:* same performance as with absolute feedback in x,v
- Decentralized integration does not give robustly stable system

→ Modifications of the control laws required to enable implementation

Filtered distributed PD-control (F-DPD)

- Control law: (Laplace domain!)

$$U_i = -\sum_{j\in\mathcal{N}_i} f_{ij}(X_i - X_j) - \sum_{j\in\mathcal{N}_i} g_{ij}(V_i - V_j) - f_0 X_i - \frac{s}{\tau s + 1} K_D X_i$$

- Low-pass filter prevents too large variations in control signal

Theorem

$$V_N^{\mathrm{F-DPD}} = \frac{1}{2N}\sum_{n=2}^{N} \frac{1}{(f_0 + f\lambda_n)\left(g\lambda_n + \frac{K_D(\tau g\lambda_n + 1)}{\tau^2(f_0+f\lambda_n)+\tau g\lambda_n+1}\right)}$$

For any positive K_D and τ, $V_N^{\mathrm{F-DPD}}$ is *uniformly bounded* in N for any network:

$$0 < V_N^{\mathrm{F-DPD}} < \frac{\tau^2 f_0 + 1}{2 f_0 K_D}$$

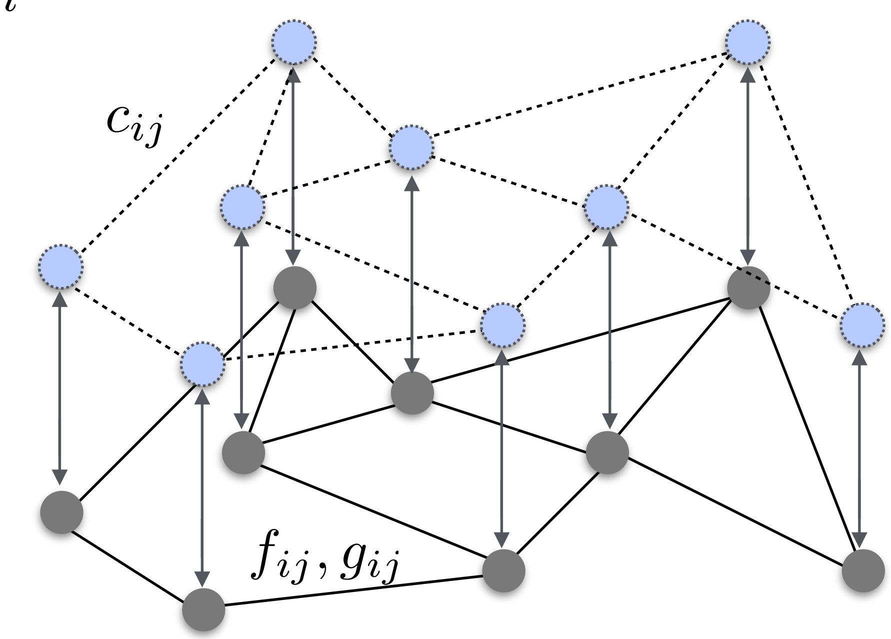
- Higher order filters give same result
- Theoretical performance best if filter constant $\tau = 0$

Distributed averaging PI-control (DAPI) 1(2)

- Control law:

$$u_i = -\sum_{j \in \mathcal{N}_i} f_{ij}(x_i - x_j) - \sum_{j \in \mathcal{N}_i} g_{ij}(v_i - v_j) - g_0 v_i - K_I z_i$$

$$\dot{z}_i = -v_i - \sum_{j \in \mathcal{N}_i} c_{ij}(z_i - z_j)$$



- Distributed averaging filter prevents de-stabilizing drift by aligning integral state
- Proposed in power system context (secondary frequency control)

Theorem

Assume uniform ratios c_{ij}/f_{ij} , so $\mathcal{L}_c = c\mathcal{L}$, then

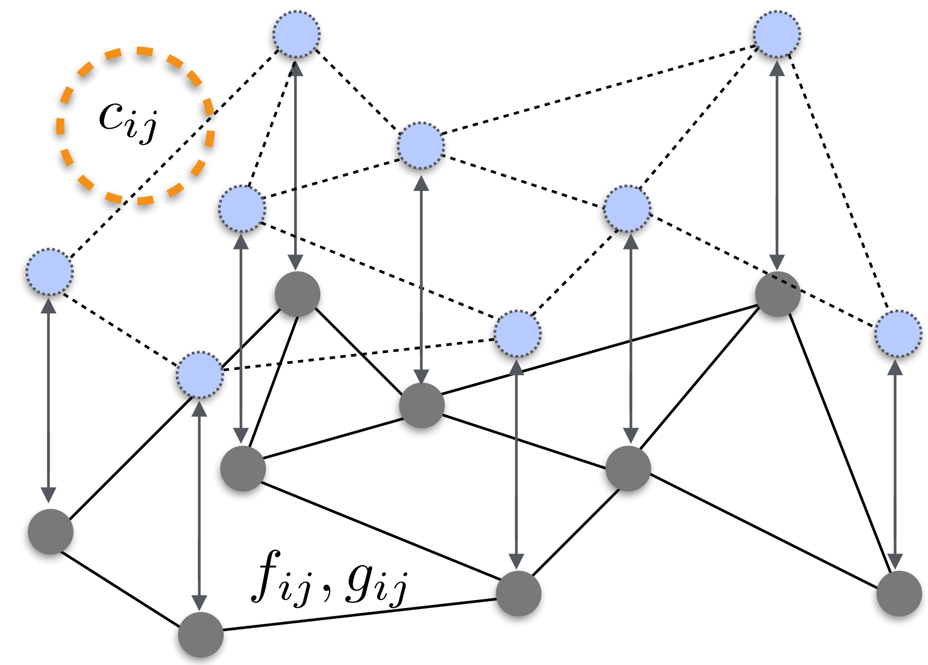
$$V_N^{\mathrm{DAPI}} = \frac{1}{2N} \sum_{n=2}^{N} \frac{1}{fg\lambda_n^2 + \frac{K_I f(g_0 + \lambda_n(c+g)) + g_0 f \lambda_n (c^2 \lambda_n + f + c g_0))}{f + c g_0 + c\lambda_n(c+g)}}.$$

For any positive and finite K_I and c, V_N^{DAPI} is *uniformly bounded* in N:

$$0 < V_N^{\mathrm{DAPI}} < \frac{f + c g_0}{2 K_I f g_0}.$$

Distributed averaging PI-control (DAPI) 2(2)

- Design of distributed averaging filter affects performance
  - $c \to 0 \Rightarrow \;\; z_i \approx \int_0^t v_i(\tau) d\tau = x_i$
  - $c \to \infty \Rightarrow$ same perf. as w/o PI control
  - In some cases, optimal $c^\star > 0$

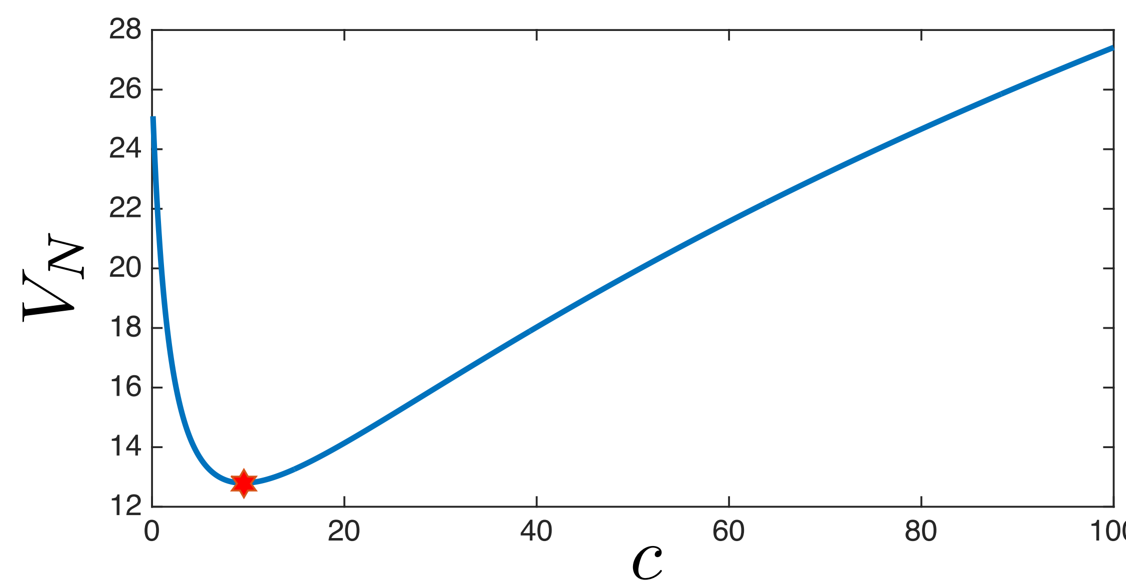


Corollary (optimal distr. averaging)

The optimal gain $c^\star > 0$ if

$$f > \frac{1}{\lambda_n}(g\lambda_n + g_0)^2,$$

for all $n = 2, \ldots, N.$



- For insights to optimal topology, see X. Wu et al. (ACC, 2016), D. Deka et al. (ACC, 2017)

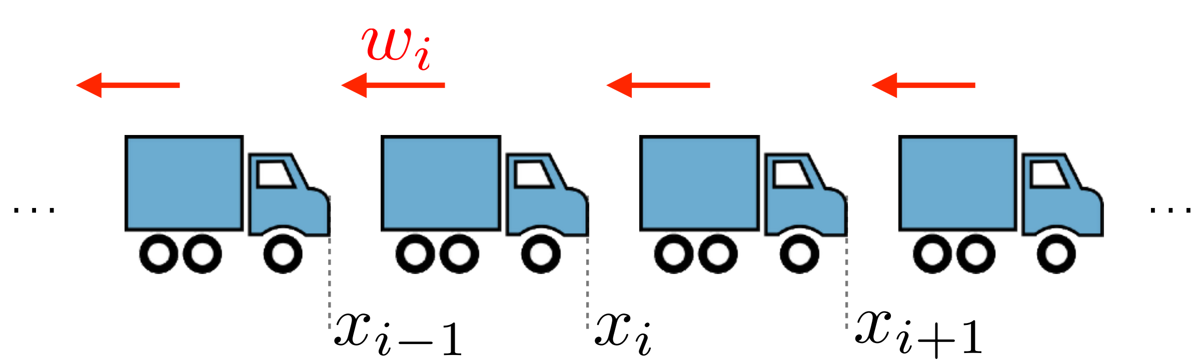
Summary: PI and PD control can relax performance limitations

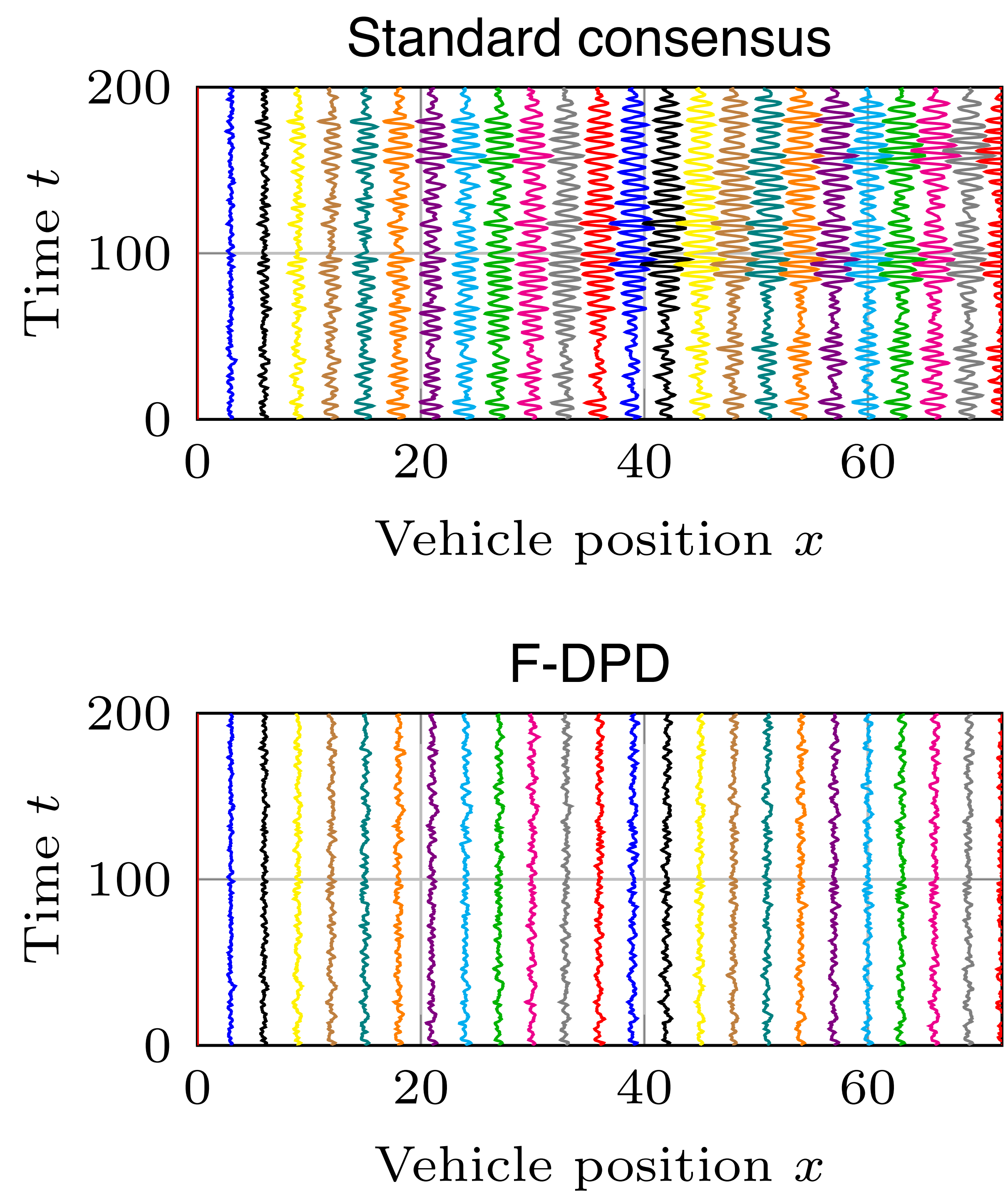
	Relative x, relative v	Relative x, absolute v Absolute x, relative v	Absolute x, absolute v
Standard consensus	$V_N \sim \frac{1}{\beta^2} \begin{cases} N^3 & d=1 \\ N & d=2 \\ N^{1/3} & d=3 \\ \log N & d=4 \\ 1 & d \geq 5 \end{cases}$	$V_N \sim \frac{1}{\beta} \begin{cases} N & d=1 \\ \log N & d=2 \\ 1 & d \geq 3 \end{cases}$	$V_N \sim \frac{1}{\beta}$
F-DPD, DAPI	N/A	$V_N \sim \frac{1}{\beta}$	$V_N \sim \frac{1}{\beta}$

(β paramater reflecting control effort, d lattice dimension)

Example 1: F-DPD in vehicular formation

- Assume no speedometer, but position is known
- Compare standard protocol to F-DPD

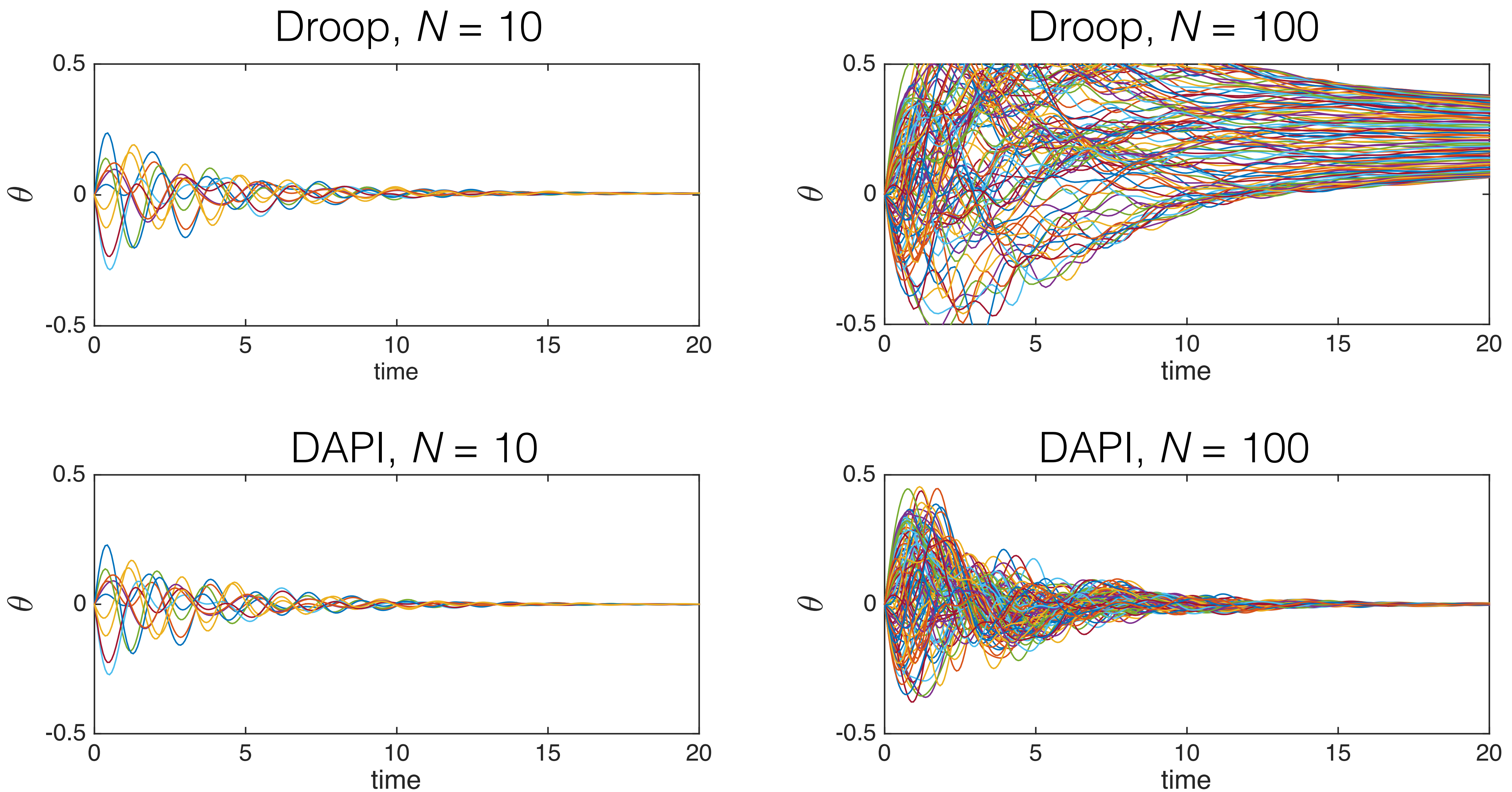




Subset of 100 vehicle platoon, simulated under white noise disturbance

Example 2: DAPI in frequency control

- In power networks, frequency ω_i can be measured, but measurement of phase θ_i requires phasor measurement unit (PMU)
- DAPI improves performance and scalability, +eliminates stationary error



Simulation of synchronization transient in radial network with N=10 and N=100 nodes

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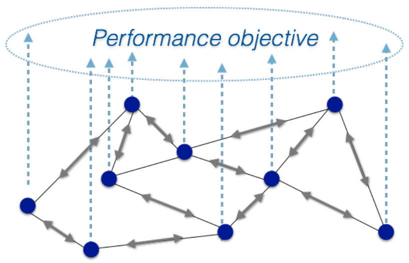

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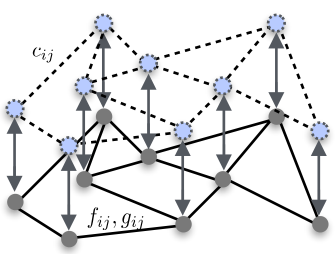
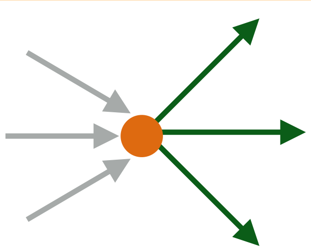


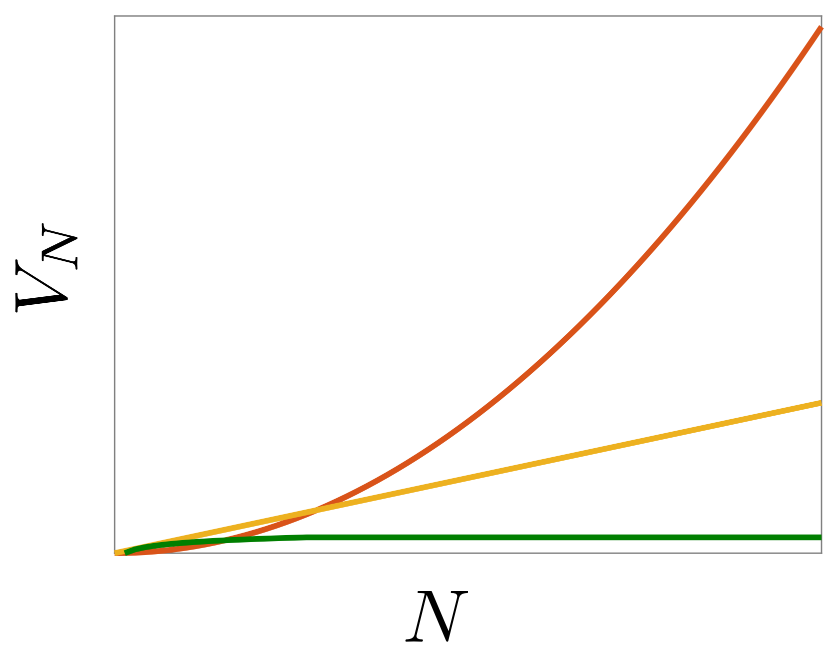
Distributed PI and PD control

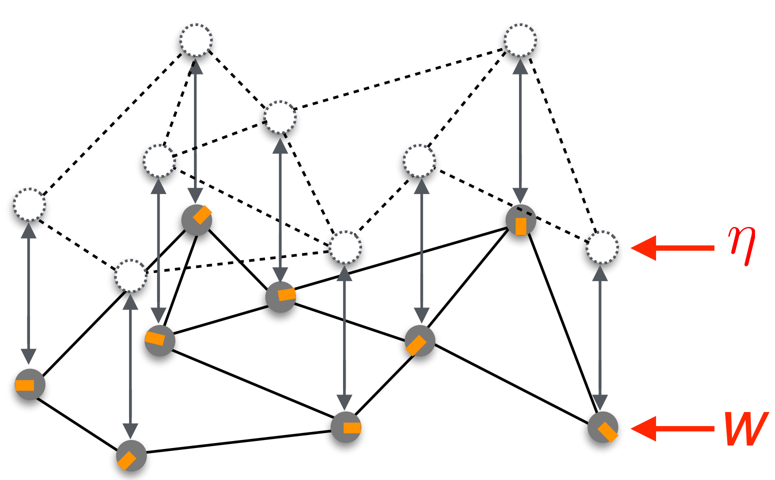


Conclusions and future work

Ongoing and future work

- Can scalings at all be improved without absolute measurements?
- Issues with measurement noise and bias
- Further applications in power networks:
  - Scalability of frequency control
  - Use of PMUs





Thank you!

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